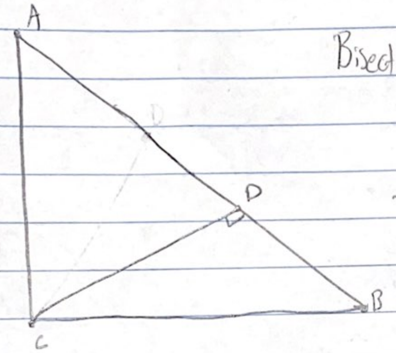
Jacob Mclane

12\6\21

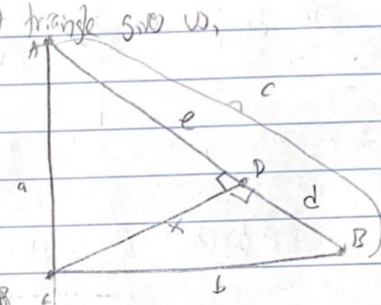
Exam 2

1. Pythagorean Theorem Proofs:

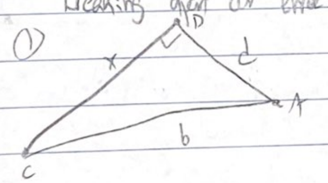
(a.) Similar Triangles

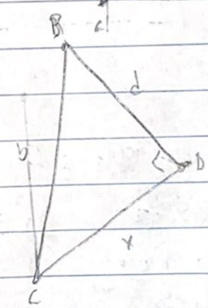


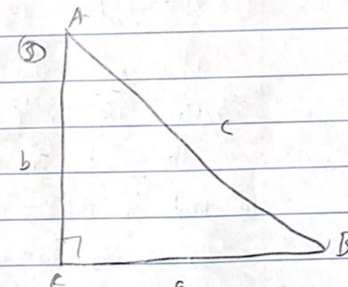
Bisecting a right triangle gives us,



Breaking apart our three triangles,







So, $\triangle ABC$ also called triangle 3, is congruent to $\triangle ACD$ (triangle 1)

$\angle CAB = \angle CAD$ and $\angle CDA = BCA = 90°$

So, $\triangle ABC \sim \triangle ACD$

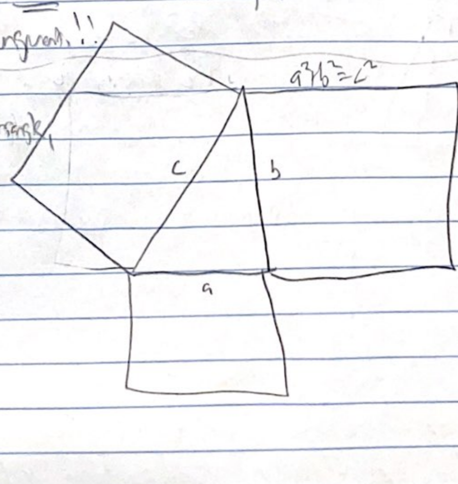
And similarly, $\triangle ABC \sim \triangle CBD$ because $\angle CBA = \angle CBD$ and $\angle ACB = \angle BDC = 90°$

Since if a triangle shares two similar angles, they are similar.

If $\triangle ABC \sim \triangle ACD$ and $\triangle ABC \sim \triangle CBC$, then $\triangle ACE \sim \triangle CBF$

Thus, they are congruent!!

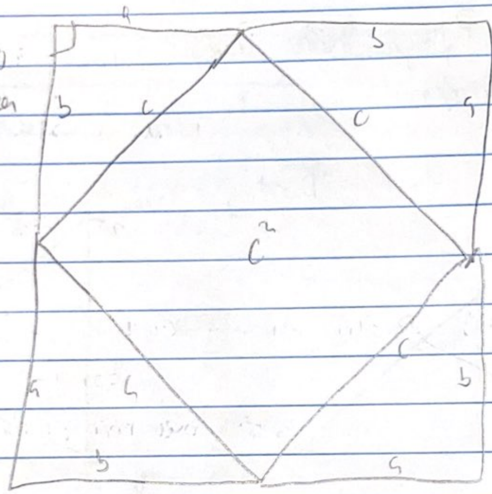
(b.) Take a right triangle,



12/6/21

Take $c^2 \rightarrow$

Now, the size of our triangles has changed because they have been squared and split in half, but the proof is shown here ⟶



(2) $\sqrt[7]{3}$ Irrational

Proof by Contradiction:

Suppose $\sqrt[7]{3}$ is rational, then $\sqrt[7]{3} = \frac{m}{n}$ for some $m,n \in \mathbb{Z}$, m,n are coprime.

Then, $\sqrt[7]{3} = \frac{m}{n} \rightarrow 3 = \frac{m^7}{n^7} = 3n^7 = m^7$

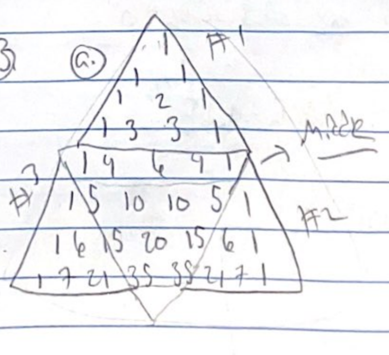
m^7 must be divisible by 3. However, if m^7 is divisible by 3, then so is m.

So we say, $3n^7 = (3k)^7 \rightarrow 3n^7 = (3)^7 \cdot k^7 = n^7 = 3^6 \cdot k^7$

We have arrived at a contradiction, n^7 must also be divisible by 3 which means m,n are not coprime.

Thus $\sqrt[7]{3}$ is irrational!

(3) (a.)



$\left(\frac{n!}{r!(n-r)!}\right)$

(b.) The Feigenbaum constant is the ratio of a bifurcation diagram on a non-linear map.

It expresses the limiting ratio of a bifurcation interval between periods.

12/6/21

(4) a. A convex polyhedron whose faces are congruent and every vertex has the same number of faces meet at it.

b. $V-E+F=2$ → Every edge has two vertices, so for V vertices we have $a \cdot V$ number of edges. Since every edge has 2 vertices then $2E = aV$ so $\boxed{V=\frac{2E}{a}}$

c. Every face has b edges around it, with F faces we have $F \cdot b$ edges. Every edge has 2 faces, so we say $2E = b \cdot F \implies F=\frac{2E}{b}$?

d. First case $\boxed{F=\frac{2E}{b}}$

$(a,b) = (3,3)$: $V=4, E=6, F=4$ → Tetrahedron $\frac{12}{3}=4$

$= (3,4)$: $F=6, E=12$ → Hexahedron $\frac{24}{4}=6$

$= (3,5)$: $F=12, E=30$ → Dodecahedron $\frac{60}{5}=12$

$= (4,3)$: $F=8, E=12$ → Octahedron $\frac{24}{3}=8$

$= (5,3)$: $F=20, E=30$ → Icosahedron $\frac{60}{3}=20$

(5) We can say that the left cosets of H in G form a partition of G. This is because two elements in G, x and y, are equivalent if $h \in H$ such that $x = yh$.

Every left coset aH has same cardinality as H

$x \to ax$ defines $H \to aH$

The number of left cosets of H in G is therefore an integer because it partitions the group into equal size.

(6.) It is called the Cauchy-Riemann.

I forgot what is special ..

(7.) William Rowan Hamilton, he lived in Dublin.

(8.) His formula was: $A=\sqrt{s(s-a)(s-b)(s-c)}$ for the area of a triangle, he lived in the first century AD.

(9.) Newton studied at Cambridge, his teacher was Isaac Barrow.

Newton left Cambridge to become master of the mint.

12/6/21

16. Leibnitz was born in Leipzig, he spent most of his life in Hanover. He served George I as a duke until he became King

(17.) (a) $\frac{2}{\pi} = \cos\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{8}\right) \cdot \cos\left(\frac{\pi}{16}\right) \cdots$

(b.) John Neper and Henry Briggs

(a)
(18.) A Eulerian path is a path that visits every edge once but does not start and end in the same place.

(b.) If two vertices have odd degree.

(c.) Each vertex needs an entry point and exit point for a Eulerian path. Now if there are many vertices with odd degree, you cannot enter and exit each vertex once you will have to touch the edge again.