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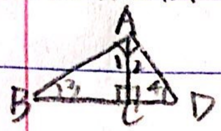
1. Proofs of Pythagorean theorem

(i) Similar Triangle



$\because \angle 4 + \angle 3 = 90^\circ$

$\angle 2 + \angle 4 = 90^\circ$

$\angle 2 = \angle 3$

$\angle ACD = \angle BAD \Rightarrow \triangle ACD$ similar to $\triangle ABD$

$AD/BD = \frac{CD}{AD}$

Thus $\frac{BC}{AC} = \frac{AC}{CD}$,

$\angle 1 + \angle 3 = 90^\circ$

$\angle 3 + \angle 4 = 90^\circ \Rightarrow \angle 1 = \angle 4$,

$\angle ACB = \angle BAD = 90^\circ$, $\triangle ABC$ is similar to $\triangle ABD$

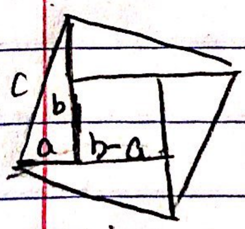
$AB/BD = \frac{BC}{AB}$

$AD^2 = BD \cdot CD \quad AB^2 = BD \cdot BC$

$AD^2 + AB^2 = BD \cdot (CD + BC) = BD^2$

Proved

(ii)



$S(\text{square}) = (b-a)^2 + 4\,ab/2 = (b-a)^2 + 2ab$

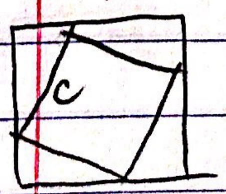
$= a^2 + b^2$

The square with side $c = c^2$

$(b+a)^2 = c^2 + 4\,ab/2 = c^2 + 2ab$

$c^2 = (b+a)^2 - 2ab = a^2 + b^2$

Proved



2. Prove that $\sqrt[7]{3}$ is irrational

Assume let $\sqrt[7]{3}$ is rational

$\sqrt[7]{3} = q/p$ (there is no prime factor in p and q

$3 = q^7/p^7$

$q^7 = 3p^7$

3 is a prime number $q^7$ could divide by 3

let $q = 3a$

$(3a)^7 = 3p^7$

$3^7 \cdot a^7 = 3p^7$

$3^6 \cdot a^7 = p^7$

$p^7$ could divide by $3^6$, $p^7$ could also divide by 3

$\Rightarrow$ p and q have a common factor 3, which contradict to the assumption, proved

3. (a)

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

(b) Feigenbaum constants are a pair of real constant

$\delta$ is the geometric approach to the bifurcation parameter to its limiting value

$$\lim_{k\to\infty} \mu_\infty - \mu_k = \frac{T}{\delta^k}$$

$$\delta = \lim_{n\to\infty} \frac{\mu_{n+1} - \mu_n}{\mu_{n+2} - \mu_{n+1}}$$

4. A (a) Patonic solid is a convex regular polyhedrin in 3D space

(b) $2E = aV$ (every edges belong to the vertices)

$2E = bF$ (every face has b edges

$V = 2E/a$, $F = 2E/b$

(c)

5. Consider the group $U(n)$, the set of all units modulo $n$.

$U(n)$ is even when $n > 2$

Suppose $n > 2$

The number can be written as $1 = n - (n-1)$

Also $\gcd(n-1, n) = 1$

$n$ and $n-1$ are both prime

$\Rightarrow n-1 \in U(n)$ (less than $n$ and prime to $n$)

$n-1 \not\equiv 1 \pmod{n}$ and $(n-1)^2 \equiv 1 \pmod{n}$

the order of $(n-1)$ is $2$

Since the order of elements divides the order of $G$

$2$ could divide the order of $U(n)$

$U(n)$ is even when $n > 2$

For Lagrange theorem, the order of each element of the group $G$ divides the order of $G$

6. Cauchy-Riemann equations

This implies u and v are differentiable

$u(x,y)+iv(x,y)$ are complex differentiable. when u and iv satisfy the equation, it must be differentiable

7. William Rowan Hamilton

Dublin

8. Area $= \sqrt{s(s-a)(s-b)(s-c)}$ for $\triangle abc$ has semi parameter s

AD 62.

9. Trinity College, Cambridge.

Isaac Barrow

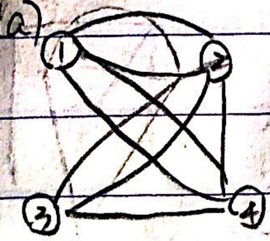
He obtained the BA degree.

10. Leipzig, Hanover, King George

11 (a) $\frac{2}{\pi} = \frac{\sqrt{2}}{2} \frac{\sqrt{2+\sqrt{2}}}{2} \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$

(b) John Napier, Joost Burgi

12. (a)

A 


$$\begin{bmatrix} 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix}$$

(b) It will be even edges on the vertex

(c) If Euler path exist, it travels each edge of a graph exactly once, it arrives at every vertex except starting and end. It will be an even edges occured on the vertex