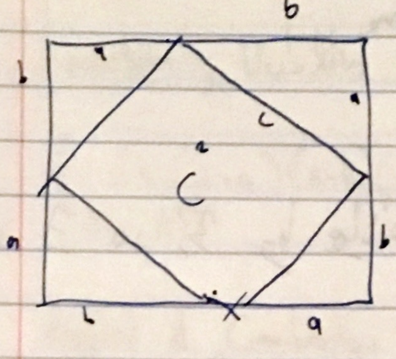
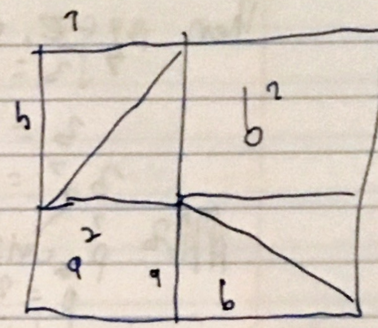
1. Rearrangement:

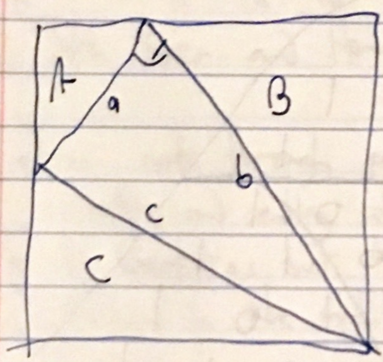


can be rearranged to be



outer square area same, triangles area same, so the rest also has the same area — $c^2 = a^2 + b^2$

Similar triangles:



A, B, C similar triangles

2. for sake of contradiction suppose $\sqrt[7]{3}$ is rational.
Then $\exists\, p,q \in \mathbb{Z}$, $q \neq 0$, where p,q coprime,

$$\sqrt[7]{3} = p/q$$
$$3 = p^7/q^7$$
$$3q^7 = p^7$$

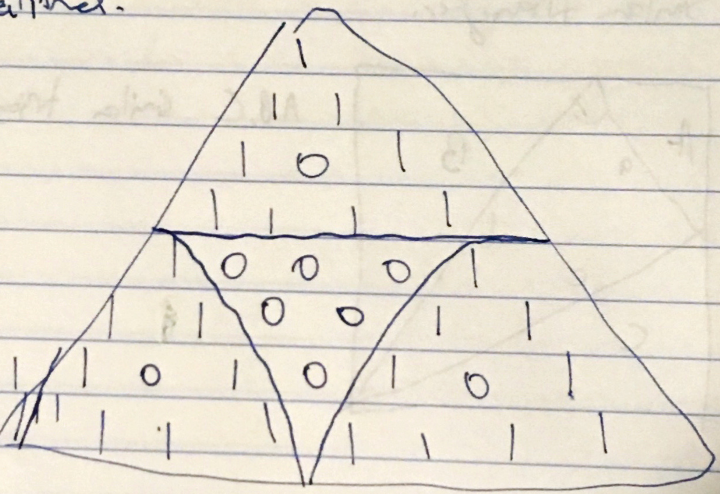
p^7 divisible by 3 iff p divisible by 3, so $\exists\, m \in \mathbb{Z}$,

$$p = 3m$$
$$3q^7 = (3m)^7$$
$$q^7 = 3^6 m^7$$

So q^7 divisible by 3, so q divisible by 3.
But p,q coprimes that are both divisible by 3 – this is absurd.
So $\sqrt[7]{3}$ is irrational.

3. a)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1



b) constant: $\lim_{k \to \infty} \frac{r_{k+1} - r_k}{r_k - r_{k-1}}$

~~k is reproduction constant of map $x_{n+1} = kx_n(1-x_n)$ ... for $k<3$, the population will stabilize towards some number. r_k is the~~

k is the reproduction constant of map $x_{n+1} = kx_n(1-x_n)$.
r_n is the transition from 2^{k-1} to time 2^k.

4. a) Platonic solid – solid/polyhedron with congruent faces, where each face has sides of same length separated by same angle, where faces meet at vertices with same degree

b) a = vertex degree
b = edges per face

two faces share 1 edge, b edges per face,

$$F = \frac{2E}{b}$$

two vertices share 1 edge, a edges per vertex,

$$V = \frac{2E}{a}$$

c. $V - E + F = 2$ (are counting infinite region)

~~$V = 2 + E - F$~~ $\quad E = \frac{Fb}{2}$

$$V = \frac{2E}{a} = \frac{2}{a}\left(\frac{Fb}{2}\right) = \frac{Fb}{a}$$

$$\frac{Fb}{a} - \frac{Fb}{2} + F = 2$$

$$F\left(\frac{b}{a} - \frac{b}{2} + 1\right) = 2$$

$$\boxed{F = \frac{2}{\frac{b}{a} - \frac{b}{2} + 1}} \quad \frac{2}{\frac{2b - ba + 2a}{2a}} = \frac{4a}{2b - ba + 2a}$$

d. $3 \le a, b \le 5$

(a,b) = (3,3). $F = \frac{12}{-6}$, not integer X

(3,4) F =

(3,5)

(4,3)

(4,4)

(4,5)

(5,3)

(5,4)

(5,5)

d) $(3,3) = \frac{2}{1-\frac{3}{2}+1} = 4$, tetrahedron

$(3,4) = \frac{12}{8-12+6} = 6$, cube

$(3,5) = \frac{12}{1} = 12$, dodecahedron

$(4,3) = \frac{16}{8-12+8} = 8$, octahedron

$(4,4)$: $\frac{16}{8-16+8}$, not positive X

$(4,5)$: $\frac{16}{10-20+8}$, not positive X

$(5,3)$: $\frac{20}{6-15+10} = 20$, icosahedron

5. $|H|$ divides $|G|$

Let H be any subgroup of group G.
then it has left cosets $G = a_1 H \cup a_2 H \cup a_3 H \ldots a_n H$.
and since each of these are known to be disjoint,
$G = a_1 H + a_2 H + a_3 H \ldots a_n H$.
so $G = H(a_1 + a_2 \ldots a_3)$ a whole number. For any sets, $|A| \times |B| = |A \times B|$, so since
$a_1 + a_2 + \ldots a_3$ must have cardinality, as do all sets,
so $|G| / |H|$ is an integer.

6. Cauchy-Riemann equations

7. Hamilton, Dublin

8. $A = \sqrt{s(s-a)(s-b)(s-c)}$
100 AD

9. studied at Cambridge, teacher Isaac Barrow, first big emphasis in proof
warden of the mint

10. Leipzig, Hanover, George I

11. a) $\frac{2}{\pi} = \cos\frac{\pi}{4}\cos\frac{\pi}{8}\cos\frac{\pi}{16}\cos\frac{\pi}{32}$
b) John Napier and Henry Briggs

12. a) a route through a graph that visits each edge exactly once, starting and ending at different vertices.
b) All vertices but two must have even degree. Those two must be odd.
c) Vertices in the middle of the path are entered and exited x times — if each one uses a different edge this gives them degree $2x$ — even.
The beginning vertex is left, entered/left x times, and not returned to. This requires $1+2x$ edges — odd degree.
The end vertex is entered/left x times then entered.
This requires $2x+1$ edges — odd degree.