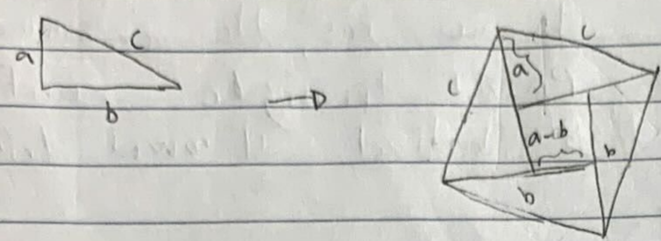
Edward Chung Midterm 2 12/6/21

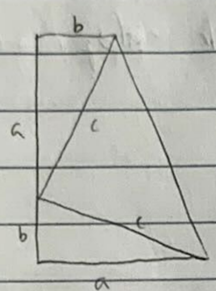
1) 2 proofs of Pythagorean Theorem

a)



$$c^2 = (a-b)^2 + 2ab$$
$$= a^2 - 2ab + b^2 + 2ab$$
$$= a^2 + b^2 \quad \square$$

b)



$$2\left(\tfrac{1}{2}ab\right) + \tfrac{1}{2}c^2 = \frac{(a+b)}{2}(a+b)$$
$$2ab + c^2 = (a+b)^2$$
$$c^2 = a^2 + 2ab + b^2 - 2ab$$
$$= a^2 + b^2 \quad \square$$

2) $\sqrt[7]{3}$ is irrational

- Assume $3^{1/7}$ is rational
- Then there is some $p/q = 3^{1/7}$ where p, q are integers and have no common factors > 1.
- Thus, $p = 3^{1/7} q \Rightarrow p^7 = 3q^7$

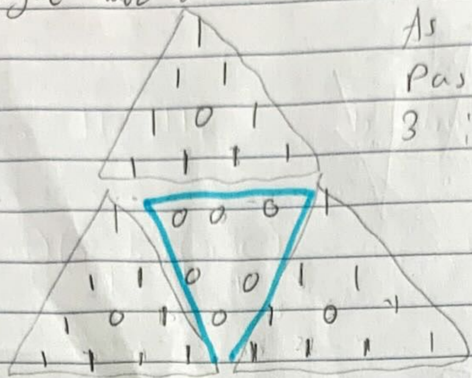
- p^7 left side is a factor of 7, but the rightside exponent of the 3 is $7k+1$.

Thus, this is a contradiction against p, q having no common factors, and false.

Thus, irrational $\square$

3) Pascal triangle mod 2

a)



As we see, the remaining Pascal triangle contains 3 identical 4-rowed triangles

b) Feigenbaum constant is a bifurcation parameter for when $x_{n+1} = a x_n (1 - x_n)$ bifurcates:

$$= \lim_{n \to \infty} \frac{r_{k-1} - r_{k-2}}{r_n - r_{n-1}} \approx 4.669\ldots$$

4) a) Platonic Solid - a polyhedron that is congruent and regular.

b) $2E = aV = bF \longrightarrow \boxed{F = \frac{2E}{b} \quad V = \frac{2E}{a}}$

c) $F + V - E = 2$

$$\rightarrow E = F + V - 2 = \frac{2E}{b} + \frac{2E}{a} - 2$$

$$2 = \frac{2E}{b} + \frac{2E}{a} - E = E\left(\frac{2}{b} + \frac{2}{a} - 1\right)$$

$$\Rightarrow E = \frac{2}{\left(\frac{2}{b} + \frac{2}{a} - 1\right)} = \boxed{\frac{2ab}{(2a + 2b - ab)}}$$

Thus, $\boxed{F = \frac{4a}{(2a + 2b - ab)}}$

d) $(a,b) = (3, 3)$

$$F = \frac{4(3)}{(2(3)+2(3)-9)} = \frac{12}{3} = 4 \text{ faces} = \underline{\text{tetrahedron}}$$

$\circ (a,b) = (3,4)$

$$F = \frac{4(3)}{(2(3)+2(4)-12)} = \frac{12}{2} = 6 \text{ faces} = \underline{\text{cube}}$$

$\circ (a,b) = (3,5)$

$$F = \frac{4(3)}{(2(3)+2(5)-15)} = \frac{12}{1} = 12 \text{ faces} = \underline{\text{dodecahedron}}$$

$\circ (a,b) = (4,3)$

$$F = \frac{4(4)}{(2(4)+2(3)-12)} = \frac{16}{2} = 8 \text{ faces} = \underline{\text{octahedron}}$$

$\circ (a,b) = (5,3)$

$$F = \frac{4(5)}{(2(5)+2(3)-15)} = \frac{20}{1} = 20 \text{ faces} = \underline{\text{icasohedron}}$$

5)

6) Cauchy-Riemann Function. It's special because

7) William Rowan Hamilton. Berlin

8) $A = \sqrt{s(s-a)(s-b)(s-c)}$, first century.

9) Cambridge University, England. Barrow. He gave up his position so Newton could have it instead.
Master of Mint

10) Leipzig. Court of Hanover. George I

11)
a) $\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \ldots$

b) Briggs and Napier

12) a) Eulerian Path - a path that starts at a vertex, and visits every edge exactly once.

b) Every vertex must have an even number of extending edges, except two (these may be odd)

c) The two odd vertices signify the start/end. Otherwords, each edge goes to a vertex, and thus must leave the vertex as well. This relationship comes in a pair, in & out, and thus must be even.