Exam II

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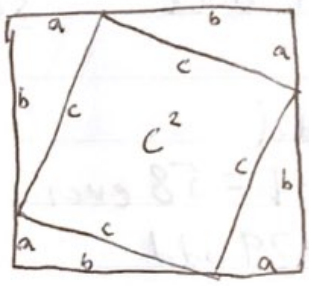
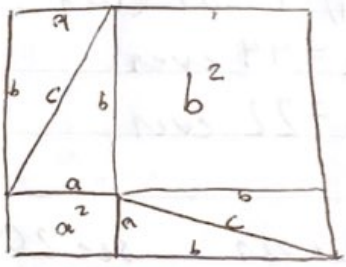
12/6

1. Rearrangement Proof:

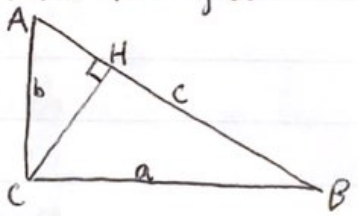
$c^2 = a^2 + b^2$



(not drawn to scale)



Proof using similar triangles:



$BC/AB = BH/BC$, $AC/AB = AH/AC$

$BC^2 + AC^2 = AB^2$

OR

$a^2 + b^2 = c^2$

2. Proof by contradiction:

if $\sqrt[7]{3}$ is rational, $\sqrt[7]{3} = a/b \Rightarrow 3 = a^7/b^7$

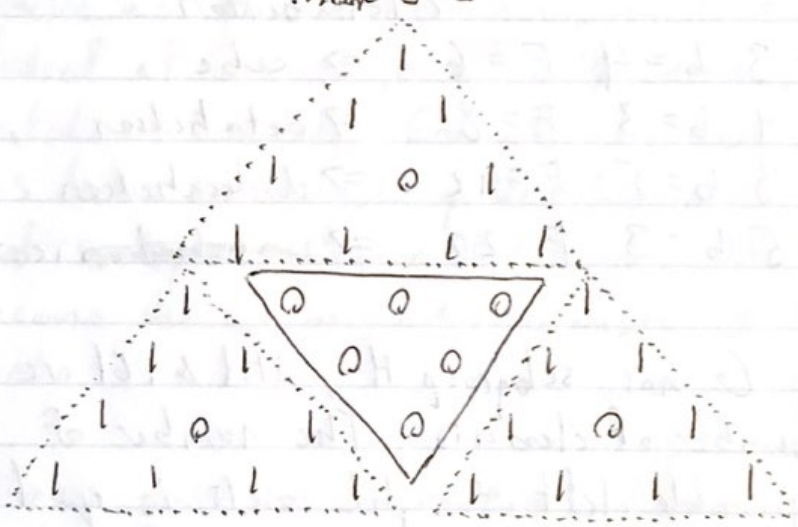
$\Rightarrow 3b^7 = a^7$

Since a & b have a GCD of 1, they must both be perfect 7th powers. However, if a^7 is a perfect power, $3b^7$ can not be. Therefore, CONTRADICTION!

3. a)

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
    1   6   15  20  15  6   1
  1   7   21  35  35  21  7   1
```

mod 2 =



3 identical triangles of form:

```
    1
  1   1
1   0   1
1   1   1   1
```

b) The Feigenbaum constant is the ratio of sequential bifurcation intervals as the period doubles.

4. a) A platonic solid is a regular, convex polyhedron in 3 dimensional Euclidean space. All faces are identical, perfect polygons.

b) $V = 4b/4-(b-2)(a-2)$ $\quad V = 2E/a$

$F = 4a/4-(b-2)(a-2)$ OR $F = 2E/b$

$E = 2ab/4-(b-2)(a-2)$ $\quad E = bF/2 = aV/2$

c) see part b

d) $a = 3 \quad b = 3 \quad F = 4$

$\Rightarrow$ tetrahedron

$a = 3 \quad b = 4 \quad F = 6 \quad \Rightarrow$ cube

$a = 4 \quad b = 3 \quad F = 8 \quad \Rightarrow$ octahedron

$a = 3 \quad b = 5 \quad F = 12 \quad \Rightarrow$ dodecahedron

$a = 5 \quad b = 3 \quad F = 20 \quad \Rightarrow$ ~~icosahedron~~ icosahedron

5. Group G has subgroup H, $|H|$ & $|G|$ are the number of elements. The number of distinguishable left or right cosets is equal to $|G|/|H|$. Cosets of this type have the property of cardinality, and H is composed by a left coset & a right coset. Therefore, $|G|/|H|$ must be an integer.

6. The Cauchy-Riemann Equations

$u(x,y) + iv(x,y)$ &

- has partial derivatives u & v

- implicates that $u(x,y) + iv(x,y)$ has a complex derivative

7. William Rowan Hamilton ; Dublin

8. $A = \sqrt{s(s-a)(s-b)(s-c)}$
He lived in 62 A.D, so the 1st century

9. Newton was born in Lincolnshire, England. He studied at Cambridge (Trinity College, located in Cambridge, England) His teacher was Isaac Barrow, who yielded Lucasian professorship to Newton. After loving, Newton became the warden, and later master, of the mint.

10. Leibniz was born in Leipzig and spent most of his life near the court of Hanover. King George I was once his employer.

11. a) $\frac{2}{\pi} = \cos(\frac{\pi}{4})\cos(\frac{\pi}{8})\cos(\frac{\pi}{16})\cos(\frac{\pi}{32})\ldots$
b) John Neper and Henry Briggs

12. a) A Eulerian Path is a Eulerian Cycle, or a way of visiting all edges of a graph exactly

once while starting at the same vertex, except it ~~may~~ ends at another vertex.

b) For a Eulerian Path to exist, there must be 2 vertices w/ odd degree.

c) This is necessary since the construction of a Eulerian Cycle requires vertices to gain adjacent edges 2 at a time... path reduces just 1 edge... out of time!