1)

Proof 1: We can start by taking a square and inscribing within it another square rotated 45 degrees. Each side of the larger square will have two lengths a and b such that a+b is the length of that side. By simple observation one can see that the area of the middle square is equal to the sum of the squares of a and b

Proof 2: We can construct similar triangles by bisecting the hypotenuse of a right triangle at an angle theta to the perpendicular. Using properties of similar triangles, the Pythagorean theorem can be derived.

2) Let us assume that $\sqrt[7]{3}$ is rational. Therefore $\sqrt[7]{3}$ = p/q for some number p and some number q. We can then manipulate this to say 3q^7 = p^7; however, this produces a contradiction as q^7 and p^7 should not have any common factors.

3a.

Pascal's triangle normal

1							
1	1						
1	2	1					
1	3	3	1				
1	4	6	4	1			
1	5	10	10	5	1		
1	6	15	20	15	6	1	
1	7	21	35	35	21	7	1

Pascal's mod 2

1							
1	1						
1	0	1					
1	1	1	1				
1	0	0	0	1			
1	1	0	0	1	1		
1	0	1	0	1	0	1	
1	1	1	1	1	1	1	1

The center 0 portion is yellow, and the similar triangles are in cyan, magenta, and green

3b. The Feigenbaum's constant

4a. A Platonic solid is a solid with faces that consist of regular polygons

4b. F= E/b by definition

We know that V-E+F=2 by Euler's identity. Subbing the first equation we found for F, we find that

V= 2- [(E-Eb)/b]

4c. We know that E=a/2 by definition. Therefore, we can say that F=(a)/(2b)

4d.

5) The number of left cosets for each subgroup will be equal and therefore Lagrange's theorem will remain true for all groups G which have a subgroup H

6) What is the name of the following famous equation-pair? ux = vy , uy = −vx , or, in fuller notation ∂u ∂x = ∂v ∂y , ∂u ∂y = − ∂v ∂x . What is special about the function u(x, y)+iv(x, y) where u(x, y), v(x, y) satisfy the above system of two equations?

The name of the equation-pair is the Cauchy-Riemann equations. The special thing about the function that satisfies it is that it maps the xy plane to the uv plane and therefore can transform surfaces between the xy plane and the uv plane.

7) Who discovered the quaternions? What city did that person live in?

Hamilton, who lived in Dublin, discovered the quaternions

8) What is Heron's formula, what century did Heron live in?

Heron's formula is a formula for the area A of a triangle when the lengths of the sides a,b, and c are known. The formula is $A = \sqrt[2]{s(s-a)(s-b)(s-c)}$ and Heron lived in the first century AD.

9) Where did Isaac Newton study? Who was his teacher? What unusual action did that teacher do? What was Newton's position after he left Cambridge?

Isaac Newton studied at Cambridge. His teacher was Isaac Barrow and the unusual thing he did was to give up the professorship to Newton. After he left Cambridge, Newton's position was warden of the mint.

10) In what city was Leibnitz born? Where did he spend most of his life? What King of England was once the employer of Leibnitz?.

Leibnitz was born in Leipzig but spent most of his life in Hanover. George I was once the employer of Leibnitz.

11. (10 points total)

(a) (5 points) State Vi`ete's infinite product for 2/π

$2/\pi = \cos(\frac{\pi}{4}) \cos(\frac{\pi}{8}) \cos(\frac{\pi}{16}) \cos(\frac{\pi}{32})\ldots$

b) (5 points) State the names of two people who initiated the use of logarithms

John Napier and Henry Briggs

12. (10 points altogether)

(a) (3 points) Define a Eulerian path in a graph.

An Eulerian path is a path in a graph G that traverses every edge in G without repeating any edges

(b) (3 points) State the necessary condition for a graph to have a Eulerian path

A graph G must have no more than 2 vertices of odd degree in order to have an Eulerian path

(c) (4 points) Prove (or explain in your own words) why the condition in (b) is necessary

Each edge connects 2 vertices. Therefore, when a path traverses an edge, it enters at one edge and exits at another. Therefore, to traverse each edge without repetition, we can only have 2 vertices from which the path exits or at which it terminates. These vertices are the beginning and ending vertices of the Eulerian path.