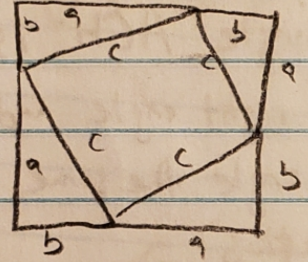
Midterm 2   Abdul-Ahed Butt

1) First Proof

We take a $(a+b) \times (a+b)$ square that has 4 righ angled triangles with base a and height b. These triangles are arranged so that there is a rectangle in the middle where the square is a $c \times c$.



The area of the square is c^2.

Each of the 4 triangles here area $\frac{(a)(b)}{2}$. This makes the total area of the whole entire square $c^2 + 4\left(\frac{(a)(b)}{2}\right) = c^2 + 2ab$.

Based on the fact this is a $(a+b) \times (a+b)$ square, we say the area of the whole entire square is $(a+b)^2 = a^2 + 2ab + b^2$

With that,

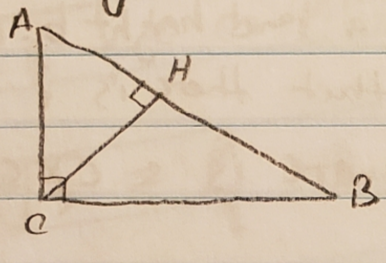
$$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$$

$$\Rightarrow a^2 + b^2 = c^2$$

Second Proof

we can use proportional sides of 2 similar triangles.

we say ABC is a right triangle.



H divides the hypotenuse into 2 parts

The new triangle ACH is similar to ABC as both have a right angle and share the angle at A, making the 3rd angle the same for both triangles.

The similar triangles gives us the ratios

$$\frac{BC}{AB} = \frac{BH}{BC} \quad \text{and} \quad \frac{AC}{AB} = \frac{AH}{AC}$$

$$\Rightarrow \; BC^2 = AB \times BH \quad \text{and} \quad AC^2 = AB \times AH$$

Adding the two sides

$$\Rightarrow \; BC^2 + AC^2 = AB(BH) + AB(AH)$$

$$= AB(BH + AH)$$

$$= AB^2$$

$$\Rightarrow \; BC^2 + AC^2 = AB^2 \; \checkmark$$

Which is like our common $a^2 + b^2 = c^2$ Pythagorean Theorem.

2) $\sqrt[7]{3}$ is irrational

we know a rational # means you can write it as an integer fraction $\frac{p}{q}$ ~~where~~ where p & q are co-primes

$\sqrt[7]{3} = \frac{p}{q} \rightarrow 3 = \frac{p^7}{q^7} \rightarrow p^7 = 3q^7$

we say $p = 3m$ then plug-in

3
27 81 243
×3 ×3 ×3
81 243 729
729
×3
2187

$(3m)^7 = 3q^7 \rightarrow 2187m^7 = 3q^7$

$q^7 = 729m^7$

This is a contradiction as 3 is divisible by p & q.
There's a common factor between p & q
Thus p & q are not coprime, meaning it cannot be rational.
Thus, $\sqrt[7]{3}$ is irrational

4) a) Platonic Solid is any of the 5 solids whose faces are identical polygons meeting at the same 3D angles. Every face has the same size and shape.

b) $a = \#$ edges meeting vertex $b = \#$ edges surrounding face

Since every vertex has edges coming out of it, there are V vertices, then we see there are aV edges. But as every edge belongs to 2 vertices, then

$$2E = aV \text{, so } E = \frac{aV}{2} \text{ and thus } V = \frac{2E}{a}.$$

Since every face has edges around it, and there are F faces, then we see that there are bF edges. But as every edge belongs to 2 faces, then

$$2E = bF \text{ so } E = \frac{bF}{2} \text{ and } F = \frac{2E}{b}$$

Using the formula $V - E + F = 2$, plug

$$\frac{2E}{a} - E + \frac{2E}{b} = 2$$

$$E\left(\frac{2}{a} - 1 + \frac{2}{b}\right) = 2$$

Thus

$$E = \frac{2}{\frac{2}{a} - 1 + \frac{2}{b}}$$

c) similerly, we have $V-E+F=2$

so plus in $V=\frac{2E}{a}$ and $E=\frac{bF}{2}$

$$\frac{2E}{a}-\frac{bF}{2}+F=2$$

$$-\frac{bF}{2}+F=2-\frac{2E}{a}$$

$$F=2-\frac{2E}{a}+\frac{bF}{2}$$

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d) $a=3, b=3$ Tetra $F=4$

$a=3, b=4$ Hexa. $F=6$

$a=3, b=5$ Dodeca. $F=12$

$a=4, b=3$ Octa. $F=8$

$a=5, b=3$ Iscosa. $F=20$

5) We say G has m elements and H has n elements.

~~H has~~ $H = \{H_1, H_2 \ldots H_n\}$

All the elements of the subgroup H are different.

Base Case if $G = H$, then they are the same and $\frac{G}{H}$ would equal 1

Otherwise, lets take an element from G which is not from H, we will take the coset

$$gH = \{gH_1, gH_2 \ldots gH_n\}$$

The elements of the coset are different as if they were identical then it would give us a contradiction. As we have taken the coset, the elements from gH are not in H but in G.

As H is a subgroup, having g be an element of H would be invalid/contradiction, as g must be from G not H.

Getting the coset decomposition $G = H \cup gH$ would give us what we need. We can also take another element of G which is not in H or gH and do the coset decomposition and get different elements from H and gH. So on and so forth to give us

$$G = H \cup g_1H \cup g_2H \cup g_3H \ldots \cup g_{n-1}H$$

in which each subgroup H has n distinct elements multiplied by the number of distinct elements 'g', we get number of elements of G is: $m = nk \rightarrow k = \frac{m}{n}$

Thus making it an integer and showing $\frac{|G|}{|H|}$

is always an integer.

6) 'Cauchy Riemann Equations'
$u(x,y) + iv(x,y)$ is an analytic function.

7) William R. Hamilton, he lived in Dublin

8) $A = \sqrt{s(s-a)(s-b)(s-c}$
A being the area of a triangle and a, b, c are the side lengths.
From 1st Century AD.

9) Studied at Cambridge Uni., England
Barrow.
Barrow gave his career of being a professor for Newton to have it.
He was master of the Mint.

10) From Leipzig
Court of Hanover
George I

11) a) $\frac{2}{\pi} = \cos\frac{\pi}{4} \cos\frac{\pi}{8} \cos\frac{\pi}{16} \cos\frac{\pi}{32} \dots\dots$

b) Napier
Briggs

12) a) Eulerian path is a trail in a graph where it vistits all the edges of the graph (each edge exactly once) and where the start and end vertex are different.

b) Neccesary condition for Eulerian Path is that all the vertices must have even degree except for 2 which are the start and end vertex. Those are odd degree.

c) This is necessary because the starting vertex only needs an exit edge and the end vertex only needs an entry edge. If we have an even edge for the start, then in order to use all edges we will also end in the start (hence it being cycle). All the vertices inside should be even as once the are entered, through 1 edge, they leave through another 1 edge, meaning they come in pairs. If the inside was odd, then we would travel through an edge already travelled through.

must be odd as we need to exit the start and enter the end

3) a)

1

1 1

1 2 1

1 3 3 1

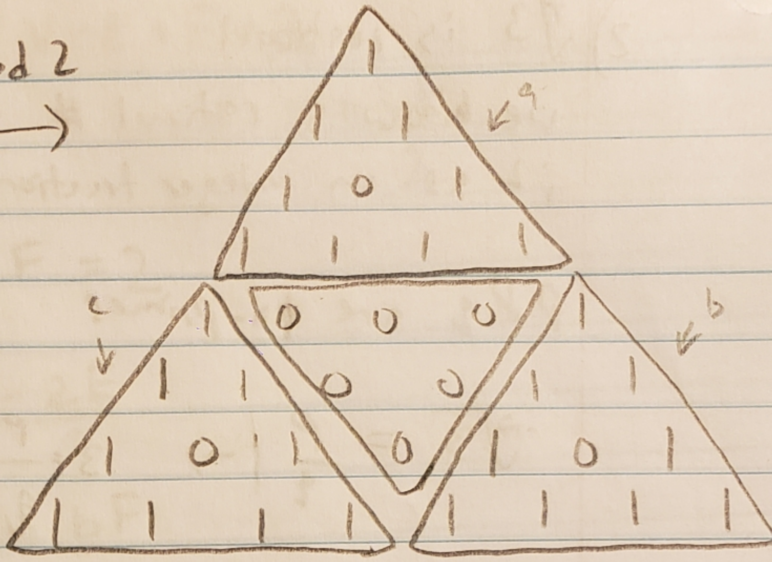
1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

$\xrightarrow{\text{mod } 2}$



We have 3 identical triangles round the middle 0 section. It has self-similarity. The triangles a, b, c are identical. each with 4 rows and one 0 in the middle.

b) we have that Feigenbaum proved the constant

$$\lim \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} \text{ is fixed constant about } \underline{4.66}$$

we see that

$x \Rightarrow ax(1-x)$ for numbers $a < 1$ always is extinct and goes to 0.

when we hit $a=3$, it is one fixed point and at $a=3.1$ the limiting behavior is $\underline{2}$ then $\underline{4}$, then $\underline{8}$ ... and so on.

It is always a power of 2. Let a_n be the bifurication parameter at which the period ~~[illegible]~~ limiting orbit changes from 2^{n-1} to 2^n