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1. Assume there have n prime number $P_1, P_2, \cdots P_n$
and $L = P_1 P_2 \cdots P_n + 1$
so L is larger than any of prime
if we need to show L is not prime, L must
divisible by at least one prime number of P_1 to P_n
but L cannot be divided by P_1 to P_n with reminder 1
$P_m \mid P_1 \cdots P_n$ and $(L - P_1 \cdots P_n) = 0$
so it is contradiction with only have n prime number
so L is also prime
there have infinity prime number

2. because 29 can not be ~~divide~~ divide by two same number
29 only can be divide by 1 and itself. so $\sqrt{29}$ is irrational

3. We know $\arctan x = \int_0^x \frac{1}{1+t^2}$
and $\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$

if $z = -t^2$

$$\frac{1}{1+t^2} = \sum_{n=0}^{\infty} (-t^2)^n = \sum_{n=0}^{\infty} (-1)^n t^{2n}$$

$$\arctan x = \int_0^x \frac{1}{1+t^2} = \int_0^x \sum_{n=0}^{\infty} (-1)^n \frac{t^{2n+1}}{2n+1} \Big|_0^x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

when $x = x^3$

$$\arctan x^3 = \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+3}}{2n+1}$$

4. a) when $k=0$

$$0(-1)(-2) = \frac{1\cdot 0\cdot -1\cdot -2}{4} = 0 \quad \text{correct}$$

when $k=1$

$$1\cdot 0\cdot -2 = 0 = 0 \quad \text{correct}$$

when $k=2$

$$0 = \frac{3\cdot 2\cdot 1\cdot 0}{4} = 0 \quad \text{correct}$$

when $k=3$

$$3\cdot 2\cdot 1 = 6 = \frac{4\cdot 3\cdot 2\cdot 1}{4} = 6 \quad \text{correct}$$

when $k=4$

$$4\cdot 3\cdot 2 = 24 = \frac{5\cdot 4\cdot 3\cdot 2}{4} = 30 = 24+6 \quad \text{correct}$$

because the formula's degree is 4, so we need use 5 cases to prove and it is wrong.

b) when $k=0$

$$0\cdot -1\cdot -2 = 0 = 0 \quad \text{correct}$$

Assume $n=k$ is correct

$$0\cdot(-1)\cdot(-2) + 1\cdot 0\cdot(-1) + \cdots + k(k-1)(k-2) = \frac{(k+1)k(k-1)(k-2)}{4}$$

when $n=k+1$

$$0\cdot(-1)(-2) + \cdots + k(k-1)(k-2) + (k+1)k\cdot(k-1) = \frac{(k+1)k(k-1)(k-2)}{4} + (k+1)(k-1)\cdot k$$

$$= \frac{(k+1)k(k-1)(k-2) + 4k(k+1)(k-1)}{4} = \frac{(k+1)(k-1)(k^2-2k+4k)}{4} = \frac{(k+1)(k-1)(k^2+2k)}{4}$$

$$= \frac{(k+2)(k+1)k(k-1)}{4} \quad \text{is correct}$$

so it is true.

5. $\frac{(7^2+1)7}{2} = 25 \cdot 7 = 175$

28	3	34	9	40	15	46	175
45	27	2	33	8	39	21	175
20	44	26	1	32	14	38	175
37	19	43	25	7	31	13	175
12	36	18	49	24	6	30	175
29	11	42	17	48	23	5	175
4	35	10	41	16	47	22	175
175	175	175	175	175	175	175	175

6. Thales 625 BC Euclid 325 BC Archimedes 287 BC Brahmagupta Febonacci

Galileo Newton Euler Gauss Zeilberger

7. Egyptian fraction is a finite sum of distinct unit fractions

$\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$

8. Greek mathematics based on geometry and adopted elements of Babylonians, Chi'ese mathematics develop real number systems,

Archimedes is traditional father of Greek mathematics

9. Philosophie Naturalis Principia Mathematica

Newton

10. $V - E + F = 2$ Euler

~~11. when n=1 $1 - \log 1 = 1$~~

~~when n=2 $1 + \frac{1}{2} - \lg 2 =$~~

~~when n=10 $1 + \cdots + \frac{1}{10} - \lg 10 = \frac{1}{2} + \cdots + \frac{1}{10} = 1.896$~~

~~12. $a_0 = 0$ $a_1 = \sin x \sin x$ 0~~

~~$a_2 = 0$ $a_4 = 0$~~

11. Let $X_n = \sum_{k=1}^{n} \frac{1}{x} - \log(n)$

$$X_{n+1} - X_n = \frac{1}{n+1} - \log\left(1 + \frac{1}{n}\right)$$

$$\leq \frac{1}{n+1} - \frac{1}{n+1} = 0$$

so X_n is decrease

$$\sum_{k=1}^{n} \frac{1}{k} - \log(n) \geq \sum_{k=1}^{n} \frac{1}{k} - \log(n) - \frac{1}{2} - \frac{1}{2n} \geq 0$$

$$X_n \geq \frac{1}{2}$$

so X_n approach $\frac{1}{2}$

12. $a_0 = 0$ $a_1 = 0$ $a_2 = 0$ $a_3 = 0$