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1. Proof. Suppose there are finite number of primes,
such that $P_1, P_2, P_3, \dots, P_n$

Consider $P = P_1 P_2 P_3 \dots P_n + 1$.

It is obvious that there is no prime numbers P_1, P_2, \dots, P_n are factors of P , because P always leave remainder 1 when divided by P_1, \dots, P_n , so P is either prime ($P > P_n$), or divisible by a prime larger than P_n , a contradiction.

2. Proof. Assume that $\sqrt{29}$ is rational,

then we can write $\sqrt{29} = \frac{m}{n}$, for $m, n \in \mathbb{Z}$.

Now take the pair of (m, n) that give the smallest $m+n$, which means m and n are coprime.

We have $29 = \frac{m^2}{n^2}$, $29n^2 = m^2$, then m can be divisible by 29 . Say $m = 29k$, where k is a positive integer, then $m^2 = 29^2 \cdot k^2$, $29 \cdot n^2 = 29^2 \cdot k^2$, $n^2 = 29k^2$, which means n also can be divisible by 29 .

It contradicts to the fact that m and n are coprime.

So $\sqrt{29}$ is irrational.

3. We know from calculus that $\arctan x = \int_0^x \frac{1}{1+t^2}$.

And geometric series formula is $\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$, $-1 < z < 1$

Replacing z by $-t^2$, we get $\frac{1}{1+t^2} = \sum_{n=0}^{\infty} (-t^2)^n = \sum_{n=0}^{\infty} (-1)^n t^{2n}$.

Hence $\arctan x = \int_0^x \frac{1}{1+t^2} = \int_0^x \sum_{n=0}^{\infty} (-1)^n t^{2n} = \sum_{n=0}^{\infty} (-1)^n \int_0^x t^{2n}$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\frac{x^{2n+1}}{2n+1} \Big|_0^x \right) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

So $\arctan x^3 = \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+3}}{2n+1}$

4. (i) Because the formula is order 3, it can be verifying 4 values.

$$n=0 : \sum_{k=0}^0 k(k-1)(k-2) = 0$$

$0 = 0$, correct .

$$n=1 : \sum_{k=0}^1 k(k-1)(k-2) = 0$$

$0 = 0$, correct .

$$n=2 : \sum_{k=0}^2 k(k-1)(k-2) = 0$$

$0 = 0$, correct .

$$n=3 : \sum_{k=0}^3 k(k-1)(k-2) = \frac{4 \cdot 3 \cdot 2 \cdot 1}{4} = 6$$

$$3 \cdot (3-1) \cdot (3-2) = 3 \cdot 2 = 6 = 6, \text{ correct.}$$

Hence it is correct for every n .

(b)

Base case : when $n=0$, $0=0$, which is true.

Then for some arbitrary number k , assume $S(n)$ is true.

$$\sum_{k=0}^n k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$$

$$\sum_{k=0}^{n+1} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4} + (n+1)n(n-1)$$

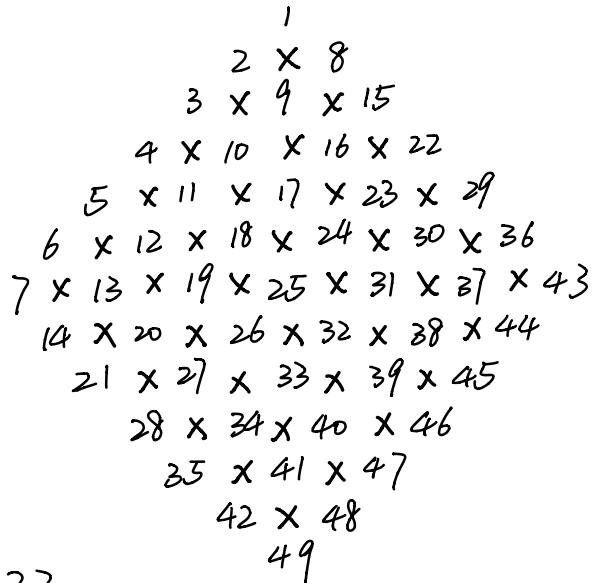
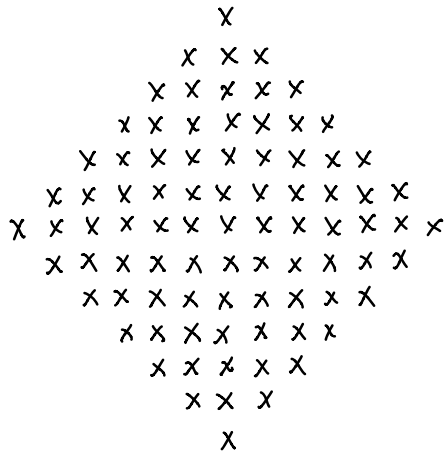
$$= \frac{(n+1)n(n-1)(n-2)}{4} + \frac{4(n+1)n(n-1)}{4}$$

$$= \frac{(n+1)n(n-1)(n-2+4)}{4}$$

$$= \frac{(n+2)(n+1)n(n-1)}{4}$$

So $S(n+1)$ is true.

5.



4	35	10	49	16	47	22
29	11	42	47	48	23	5
12	36	18	49	24	6	30
37	19	43	25	7	31	13
20	44	26	1	32	14	38
45	27	2	33	8	39	21
28	3	34	9	40	15	46

6.

Zuclid (Mid-4 century BC), Archimedes (287 BC),
 Brahmagupta (6c), Fibonacci (12c), Galileo (16c), Newton (17c),
 Thales (17c), Euler (18c), Gauss (18c), Zeilberger (20c)

$$7. \frac{5}{6} = \frac{1}{2} + \left(\frac{5}{6} - \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{3}$$

8. Most of Greek mathematics was based on geometry.

Ancient Chinese mathematics was based on real problems like trade.

Ancient Babylonian had their own system to deal with problems.

Archimedes was the traditional father of Greek mathematics.

9.

10. $F + V = E + 2$. Euler.

11. Euler's constant. Its approximate value is e .

12. $\sin(z) = z - \frac{1}{6}z^3 + \dots$

$$\sin \sin \sin(x) = \sin \sin(x) - \frac{1}{6}(\sin \sin(x))^3 + \dots$$

$$= \sin x - \frac{1}{6}\sin^3(x) - \frac{1}{6}(\sin x - \frac{1}{6}\sin^3(x))^3 + \dots$$

$$= x - \frac{1}{6}x^3 - \frac{1}{6}(x - \frac{1}{6}x^3)^3 - \frac{1}{6}(x - \frac{1}{6}x^3 - \frac{1}{6}(x - \frac{1}{6}x^3)^3)^3 + \dots$$

$$= x - \frac{1}{6}x^3 - \frac{1}{6}(x^3 - \dots) - \frac{1}{6}(x^3 - \dots) + \dots$$

$$= x - \frac{1}{2}x^3 + \dots$$

Answer: $a_0 = 0$, $a_1 = 1$, $a_2 = 0$, $a_3 = -\frac{1}{2}$