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1. Proof. Suppose there are finite number of primes, such that $p_1, p_2, p_3, \dots, p_n$

Consider $P = p_1 p_2 p_3 \cdots p_n + 1$.

It is obvious that there is no prime numbers $p_1, p_2, \cdots p_n$ are factors of P, because P alwayse leave remainder 1 when divided by $p_1, \cdots p_n$, so P is either prime ($P > p_n$), or divisble by a prime larger than p_n, a contradiction.

2. Proof. Assume that $\sqrt{29}$ is rational,

then we can write $\sqrt{29} = \frac{m}{n}$, for $m, n \in \mathbb{Z}$.

Now take the pair of (m, n) that give the smallest $m+n$, which means m and n are coprime.

We have $29 = \frac{m^2}{n^2}$, $29n^2 = m^2$, then m can be divisible by 29, say $m = 29k$, where k is a positive integer,

then $m^2 = 29^2 \cdot k^2$, $29 \cdot n^2 = 29^2 \cdot k^2$, $n^2 = 29k^2$, which means n also can be divisible by 29.

It contradicts to the fact that m and n are coprime.

So $\sqrt{29}$ is irritional.

3. We know from calculus that $\arctan x = \int_0^x \frac{1}{1+t^2}$.

And geometric series formula is $\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$, $-1<z<1$

Replacing z by $-t^2$, we get $\frac{1}{1+t^2} = \sum_{n=0}^{\infty} (-t^2)^n = \sum_{n=0}^{\infty} (-1)^n t^{2n}$.

Hence $\arctan x = \int_0^x \frac{1}{1+t^2} = \int_0^x \sum_{n=0}^{\infty} (-1)^n t^{2n} = \sum_{n=0}^{\infty} (-1)^n \int_0^x t^{2n}$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\frac{t^{2n+1}}{2n+1} \Big|_0^x \right) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

So $\arctan x^3 = \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+3}}{2n+1}$

4. (i) Because the formula is order 3, it can be vertifying 4 values.

$n=0$: $\sum_{k=0}^{0} k(k-1)(k-2) = 0$

$0=0$, correct.

$n=1$: $\sum_{k=0}^{1} k(k-1)(k-2) = 0$

$0=0$, correct.

$n=2$: $\sum_{k=0}^{2} k(k-1)(k-2) = 0$

$0=0$, correct.

$n=3$: $\sum_{k=0}^{3} k(k-1)(k-2) = \frac{4\cdot 3\cdot 2\cdot 1}{4} = 6$

$3\cdot(3-1)\cdot(3-2) = 3\cdot 2 = 6 = 6$, correct.

Hence it is correct for every n.

(b) Base case: when $n=0$, $0=0$, which is true.

Then for some arbitrary number k, assume $S(n)$ is true.

$$\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$$

$$\sum_{k=0}^{n+1} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4} + (n+1)n(n-1)$$

$$= \frac{(n+1)n(n-1)(n-2)}{4} + \frac{4(n+1)n(n-1)}{4}$$

$$= \frac{(n+1)n(n-1)(n-2+4)}{4}$$

$$= \frac{(n+2)(n+1)n(n-1)}{4}$$

So $S(n+1)$ is true.

5.

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            x
          x x x
        x x x x x
      x x x x x x x
    x x x x x x x x x
  x x x x x x x x x x x
x x x x x x x x x x x x x
  x x x x x x x x x x x
    x x x x x x x x x
      x x x x x x x
        x x x x x
          x x x
            x
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                        1
                     2  x  8
                  3  x  9  x  15
               4  x  10 x  16 x  22
            5  x  11 x  17 x  23 x  29
         6  x  12 x  18 x  24 x  30 x  36
      7  x  13 x  19 x  25 x  31 x  37 x  43
         14 x  20 x  26 x  32 x  38 x  44
            21 x  27 x  33 x  39 x  45
               28 x  34 x  40 x  46
                  35 x  41 x  47
                     42 x  48
                        49
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4	35	10	49	16	47	22
29	11	42	47	48	23	5
12	36	18	49	24	6	30
37	19	43	25	7	31	13
20	44	26	1	32	14	38
45	27	2	33	8	39	21
28	3	34	9	40	15	46

6.

Euclid (Mid-4 centry BC), Archimedes (287BC), Brahmagupta (6c). Fibonacci (12C), Gallileo (16C), Newton (17C), Thales (17C), Euler (18C), Gauss (18C), Zeilberger (20C)

7. $\frac{5}{6} = \frac{1}{2} + \left(\frac{5}{6} - \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{3}$

8. Most of Greek mathematics was based on geometry.

Ancient Chinese mathematics was based on real problems like trade.

Ancient Babylonian had their own system to deal with problems.

Archimedes was the traditional father of Greek mathematics.

9.

10. $F+V=E+2$. Euler.

11. Euler's constant. Its approximat value is e.

12. $\sin(z) = z - \frac{1}{6}z^3 + \cdots$

$$\sin\sin\sin(x) = \sin\sin(x) - \frac{1}{6}(\sin\sin(x))^3 + \cdots$$

$$= \sin x - \frac{1}{6}\sin^3(x) - \frac{1}{6}(\sin x - \frac{1}{6}\sin^3(x))^3 + \cdots$$

$$= x - \frac{1}{6}x^3 - \frac{1}{6}(x - \frac{1}{6}x^3)^3 - \frac{1}{6}(x - \frac{1}{6}x^3 - \frac{1}{6}(x - \frac{1}{6}x^3)^3)^3 + \cdots$$

$$= x - \frac{1}{6}x^3 - \frac{1}{6}(x^3 - \cdots) - \frac{1}{6}(x^3 - \cdots) + \cdots$$

$$= x - \frac{1}{2}x^3 + \cdots$$

Answer: $a_0 = 0$, $a_1 = 1$, $a_2 = 0$, $a_3 = -\frac{1}{2}$