NAME: (print!) Robert Mattox

Email DrZlinear@gmail.com as soon as I tell you (around 4:20 pm)
Subject: mt1
with an attachment called
mt1FirstLast.pdf (e.g. mt1LehonardEuler.pdf)
E-Mail address: Rgm110@ . . .edu

MATH 437 Exam I for Dr. Z.'s, Fall 2021 (Oct. 27, 2021)

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook)
Show your work! An answer without showing your work will get you zero points.
A random subset of the students will be picked for short (private!) chats via WebEx, in order to verify that there was no outside help

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

11. (out of 10)

12. (out of 10)

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total: (out of 120)

1. (10 pts.) Prove that there are infinitely many primes.

Contradiction

only finite primes

$p = p_1 \times p_2 \times p_3 \cdots p_n \times + n$

p is larger than previous numbers

since $p_1 p_2$ are are constitute allprimes

p can't be prime

so must divisible previos primes of p_m with $(1 \le m \le n)$

when divide p by p_m. we get aremaired of one

thats contraction of original stament of there are infinitive many primes.

2. (10 pts.) Prove that $\sqrt{29}$ is irrational.

contradiction proof ration if $x = a/b$, $a, b \in \mathbb{Z}$

$\sqrt{29}$ is integer

so $\sqrt{29} = a/b$

$29 = a^2/b^2$

$29b^2 = a^2$

~~$a = 2k+1$~~

so ~~m~~ ~~fist~~ $a = \cancel{2k+1}$ ~~(odd)~~ (29k) and by 29 then b must even

$29b^2 = 29^2 k^2$

$b^2 = 29k^2$

this contradiction since b is even

so $\sqrt{29}$ is irrational.

3. (10 pts) Derive (from scracth, only using geometric series, and calculus) the Taylor series around $x = 0$ of the function

$$\arctan x^3 \quad .$$

Explain all steps!

$$\sum_{n=0}^{\infty} (-1) \frac{t^{2n+}}{2n+1}$$

$x^3 \to 0$

$$\tan^{-1} x^3 = \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n+1}}{2n+1}$$

$$\tan^{-1}(x) = \frac{1}{1+x^2} = (1-x^2)^{-1}$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+3}}{2n+1}$$

$$\tan^{-1} x - \tan^{-1}(0) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$\tan^{-1} x^3 = x^3 - \frac{x^9}{3} + \frac{x^{15}}{5} - \frac{x^{21}}{5+}$$

4. (10 pts. altogether) Prove that

$$\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4} \quad ,$$

(i) (5 points): The Dr. Z. way (verifying it for sufficiently many special cases, explain how many you need)

you would need 5 since, first 3 will be zero then next two would following pattern.

(ii) (5 points): The traditional way, using **complete mathematical induction.**

$$\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)\,n(n-1)(n-2)}{4}$$

Let = 1

so that be

$$0 = 0$$

so true for 1

Let say true for any n

so now for n+1

p(n+1)

$$\sum_{k=0}^{n+1} k(k-1)(k-2) = \frac{(n+2)(n+1)(n)(n-1)}{4}$$

$$\sum_{k=0}^{n} k(k-1)(k-2) + n+1(n)(n-1) =$$

$\frac{450}{20000} = 2450/2 = 7\overline{)1225} = 175$

5. (10 points) Construct a seven by seven Magic Square.

highest number is 49

~~Euler~~ formula $\frac{49(49+1)}{2} = \frac{1225}{7} = 175$ per row
Gauss

$a+b+c+d+e+f+g = 175$
$a+$

number tricks

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

6. (10 points) Arrange the following people according to their year-of-birth, from oldest to youngest. 7 6 8 9 10 2 1 4

7 Newton, Archimedes, Gallileo, Euler, Gauss, Zeilberger, Euclid, Thales, Brahmagupta,
5 Fibonacci.

For each person, state their century of birth.

~~Brahmagupta~~(

Thales (625 BCE), Euclid 4th century BC, Archimedes (287 3rd century BC

Brahmagupta (6th century 590), Fibonacci (12 century 1170)

Gallile (1564 16th century), Newton (17 century 1643)

Euler (18th century 1707), Gauss (18 century 1787)

Zeilberger (20th century

(4) Focus the right

$$\frac{(n+2)[(n+1)\ n(n-1)]}{4} - \frac{4(n+1)(n)(n-1)}{4}$$

~~n^4~~ seperate

$$\frac{(n+2-4)\ (n+1)\ n\ (n-1)}{4}$$

$$\frac{(n-2)\ (n+1)n\ \ n-1}{4}$$

so by induction true!

(5) a b c de f g

h i j k L m n

o p q r s t u

v w x y z β ϵ

Δ q ψ λ 9 π

1

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$\frac{1}{2} = \frac{3}{6}$ $\frac{2}{6} = \frac{1}{3}$

7. (10 points). What is an Egyptian fraction? Express $\frac{5}{6}$ as an Egyptian fraction

$\frac{1}{2} + \frac{1}{3}$

8. (10 points) What is the difference between Ionian (Greek) mathematics and ancient Babylonian and Chinese mathematics? Who was the traditional father of Greek mathematics?

Ionian - how why plac iusue, more logical
Thales is father

~~Euclid father geometry~~
~~or archimedes~~

9. (10 points) What book, except for the bible, was the most reproduced and studied in the Western world? Who was its author?

Euclid Elements

10. (10 points) In a closed polyhedron, what is a relation between V, the number of vertices, E, the number of edges, and F, the number of faces? Who is it due to?

Euler

$V - E + F = 2$

11. (10 points) What is the name of the following constant: Euler constant

$$\lim_{n\to\infty}\left(\frac{1}{1}+\frac{1}{2}+\ldots+\frac{1}{n}-\log n\right) \quad .$$

What is its approximate value?

$\lim_{n\to\infty}\left(-\log n + \sum_{k=1}^{n}\frac{1}{k}\right)$

.57721

12. (10 points) Using the beginning of the famous Taylor expansion, about $x = 0$ for $\sin x$, namely

$$\sin(x) = x - \frac{1}{6}x^3 + \ldots \quad ,$$

find the beginning (up to term x^3) of the Taylor series, about $x = 0$ of

$$f(x) = \sin\sin\sin x \quad ,$$

in the form

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

Ans.:

$a_0 =$ 0; $\quad a_1 =$ x ; $\quad a_2 =$ 0 ; $\quad a_3 = \frac{x^3}{2}$.

$\sin\left(x - \frac{1}{6}x^3\right)$

$\sin\left(x - \frac{1}{6}x^3 + \ldots - \frac{x^4}{6} + \frac{x^6}{36}\right)$

$\sin(\sin x$

$~~x - \frac{1}{6}x^3 - \frac{x^4}{6} + \frac{x^6}{36} - \left(\frac{x^4}{6} - \frac{x^6}{36} + \frac{x^7}{36}\right) + x^{12}~~$

$\frac{}{36 \cdot 36}$

$\sin\sin(x)$.

$x - \frac{x^3}{6} + \frac{x^3}{6}$

$\sin\sin = x - \frac{x^3}{3} + \frac{x^3}{6} = x - \frac{x^3}{2}$