NAME: (print!) Quin Buob

Email DrZlinear@gmail.com as soon as I tell you (around 4:20 pm)
Subject: mt1
with an attachment called
mt1FirstLast.pdf (e.g. mt1LehonardEuler.pdf)
E-Mail address: qwb1@scarletmail.rutgers.edu

MATH 437 Exam I for Dr. Z.'s, Fall 2021 (Oct. 27, 2021)

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook)
Show your work! An answer without showing your work will get you zero points.
A random subset of the students will be picked for short (private!) chats via WebEx, in order to verify that there was no outside help

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

11. (out of 10)

12. (out of 10)

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total: (out of 120)

1. (10 pts.) Prove that there are infinitely many primes.

Sopposе there are finite many primes ; $P_1, P_2, P_3, \cdots P_n$

Make P a number s.t.

$$P = P_1 \times P_2 \times P_3 \times \cdots \times P_n + 1$$

When P is divided by any prime $P_1 \cdots P_n$ there is a remainder 1

There fore ther are 2 cases

Either P is a new prime or P is divisible by a new prime not in $[P_1 \cdots P_k]$

This is a contradiction there fore there must be infinitey may primes

2. (10 pts.) Prove that $\sqrt{29}$ is irrational.

Suppose $\sqrt{29} = \frac{m}{n}$ WLOG m and n are relative primes not multiples of 29

$m^2 = 29n^2$

Since m^2 is divisible by 29 then m is divisible by 29, because everything that shows up in the prime decomposition of m^2 is also present in m

$\Rightarrow m = 29r$

$(29r)^2 = 29n^2$

$29^2 r = 29n^2$

$n^2 = 29r$

By the same logic as m^2, n must be divisible by 29 b/c n^2 is divisible by 29

This is a contradiction therefore $\sqrt{29}$ must be irrational

3. (10 pts) Derive (from scracth, only using geometric series, and calculus) the Taylor series around $x = 0$ of the function

$$\arctan x^3 .$$

Explain all steps!

$\arctan(y) = \int \frac{1}{1+y^2}$

We know $\sum_{n=0}^{\infty} t^n = \frac{1}{1-t}$ $|t| < 1$

Substitute $t = -y^2$

$\sum_{n=0}^{\infty} (-y^2)^n = \frac{1}{1+y^2} = \sum_{n=0}^{\infty} (-1)^n y^{2n}$

$\arctan(y) = \int \sum_{n=0}^{\infty} (-1)^n y^{2n}$

Substitute $y = x^3$

$\arctan(x^3) = \int \sum_{n=0}^{\infty} (-1)^n (x^3)^{2n} = \int \sum_{n=0}^{\infty} (-1)^n x^{6n}$

Do term by term integration

$\arctan(x^3) = \int (1 - x^6 + x^{12} - x^{18} \ldots)$

$\arctan(x^3) = x - \frac{x^7}{7} + \frac{x^{13}}{13} - \frac{x^{19}}{19} \cdots$

$\arctan(x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+1}}{(6n+1)}$

4. (10 pts. altogether) Prove that

$$S(n)=\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4} \quad ,$$

(i) (5 points): The Dr. Z. way (verifying it for sufficiently many special cases, explain how many you need)

We have a polynomial of degree 4 therefore we need to check 5 cases because if 2 polynomials of degree d coincide at d+1 places, then they coincide everywhere

$S(0) = 0 = 0$

$S(1) = 0 + (1)(1-1)(1-2) = 0 = \frac{(1+1)(1)(1-1)(1-2)}{4} = 0$

$S(2) = 0+0+(2)(2-1)(2-2) = 0 = \frac{(2+1)(2)(2-1)(2-2)}{4} = 0$

$S(3) = 0+0+0+3(3-1)(3-2) = 6 = \frac{(3+1)(3)(3-1)(3-2)}{4} = 6$

$S(4) = 0+0+0+6+4(4-1)(4-2) = 30 = \frac{(4+1)\cancel{(4)}(4-1)(4-2)}{\cancel{4}} = 30$

These expressions are equivalent.

(ii) (5 points): The traditional way, using **complete mathematical induction**.

Prove base case

$S(0) = \sum_{k=0}^{0} k(k-1)(k-2) = 0 , \frac{(0+1)(0)(0-1)(0-2)}{4} = 0$

Show $S(n-1)$ implies $S(n)$

$S(n-1) = 0+0+0+6+30+ \ldots (n-1)\overset{(n-2)}{(n-1-1)}\overset{(n-3)}{(n-1-2)} = \frac{(n\cancel{-1}\cancel{+1})(n-1)(n-1-1)(n-1-2)}{4}$

$= \frac{n(n-1)(n-2)(n-3)}{4}$

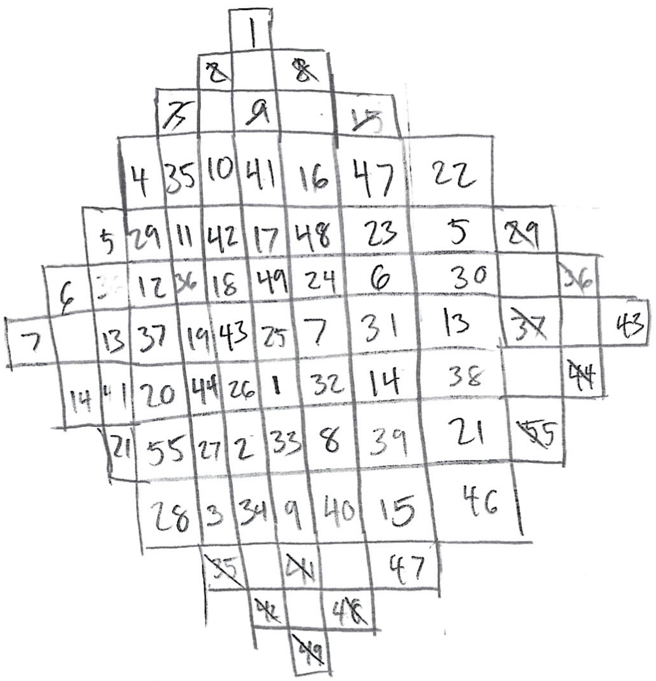
$S(n) = 0+0+0+6+30+ \ldots (n-1)(n-2)(n-3) + n(n-1)(n-2)$

$= \frac{n(n-1)(n-2)(n-3)}{4} + n(n-1)(n-2)$

$= \frac{n(n-1)(n-2)(n-3)+4n(n-1)(n-2)}{4}$

$= \frac{n(n-1)(n-2)+4(n-3)}{4}$

5. (10 points) Construct a seven by seven Magic Square.



$\Rightarrow$

$$\begin{bmatrix} 4 & 35 & 10 & 41 & 16 & 47 & 22 \\ 29 & 11 & 42 & 17 & 48 & 23 & 5 \\ 12 & 36 & 18 & 49 & 24 & 6 & 30 \\ 37 & 19 & 43 & 25 & 7 & 31 & 13 \\ 20 & 44 & 26 & 1 & 32 & 14 & 38 \\ 55 & 27 & 2 & 33 & 8 & 39 & 21 \\ 28 & 3 & 34 & 9 & 40 & 15 & 46 \end{bmatrix}$$

6. (10 points) Arrange the following people according to their year-of-birth, from oldest to youngest.

1643 287BC 1564 1701 1777 4th C BC 625 BC 590

Newton, Archimedes, Gallileo, Euler, Gauss, Zeilberger, Euclid, Thales, Brahmagupta, Fibonacci.

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For each person, state their century of birth.

Thales: 7c BC; Euclid 4th C BC, archimedes 3rd c BC, Brahmagupta 6th C. AD; Fibbonaci 12th C AD

Galileo 16th C AD; Newton 17th C AD, Euler 18th AD; Gauss 18th C AD,

Zeilberger 20th c AD

7. (10 points). What is an Egyptian fraction? Express $\frac{5}{6}$ as an Egyptian fraction

An egyptian fraction is the addition of fractions of the form $\frac{1}{n}$ where n is a whole number

$\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$

8. (10 points) What is the difference between Ionian (Greek) mathematics and ancient Babylonian and Chinese mathematics? Who was the traditional father of Greek mathematics?

Ionian Math was purely logic based, while babylonian and green math was applied and arithmetial based, no real logic applied to to it

Euclid is the father of greek math

9. (10 points) What book, except for the bible, was the most reproduced and studied in the Western world? Who was its author?

Euclids Elements

Written by Euclid

10. (10 points) In a closed polyhedron, what is a relation between V, the number of vertices, E, the number of edges, and F, the number of faces? Who is it due to?

$V + F - E = 2$

Produced by Leanard Euler

11. (10 points) What is the name of the following constant:

$$\lim_{n\to\infty}\left(\frac{1}{1}+\frac{1}{2}+\ldots+\frac{1}{n}-\log n\right) \quad .$$

What is its approximate value?

Constant of Euler

≈ 0.577216

12. (10 points) Using the beginning of the famous Taylor expansion, about $x=0$ for $\sin x$, namely

$$\sin(x) = x - \frac{1}{6}x^3 + \ldots \quad ,$$

find the beginning (up to term x^3) of the Taylor series, about $x=0$ of

$$f(x) = \sin\sin\sin x \quad ,$$

in the form

$$a_0 + a_1x + a_2x^2 + a_3x_{\cancel{3}}^{3} + \ldots$$

Ans.:

$a_0 = 0$; $\quad a_1 = 1$; $\quad a_2 = 0$; $\quad a_3 = -\frac{1}{2}$.

$g(x) = \sin x = x - \frac{1}{6}x^3$

$h(g(x)) = \sin(x - \frac{1}{6}x^3) \doteq (x - \frac{1}{6}x^3) - \frac{1}{6}(x - \frac{1}{6}x^3)^3 = x - \frac{1}{6}x^3 - \frac{1}{6}x^3 \ldots$

$s(h(g(x))) = \sin(x - \frac{1}{3}x^3) = x - \frac{1}{3}x^3 - \frac{1}{6}(x - \frac{1}{3}x^3)^3 = x - \frac{1}{3}x^3 - \frac{1}{6}x^3 \ldots$

$s(h(g(x))) = \sin(\sin(\sin x)) = x - \frac{1}{2}x^3$