EXAM I : NINA CHALGERI

① Proof by Contradiction.

Suppose that there are a finite number of primes, n, in ascending order : $p_1 < p_2 < p_3 < \dots < p_n$

- Let :

$$P = p_1 \cdot p_2 \cdot p_3 \cdot \dots \cdot p_n + 1$$

- If we divide P by p_1 then p_2 then p_3 ... all the way to p_n, it will leave us with a remainder of 1.
- This means one of two things:
  - (1) P is a prime number greater than p_n
  - (2) P is divisible by a another prime number that is larger than p_n
- This is a contradiction. There are infinitely many primes. ■

② $\sqrt{29}$ is ~~irrational~~ ~~in~~ rational.

- We can rewrite it as

$$\sqrt{29} = \frac{p}{q} \quad \text{for } p, q \in \mathbb{Z} \text{ and } p/q \text{ are in lowest terms}$$

$$\Rightarrow (\sqrt{29})^2 = \left(\frac{p}{q}\right)^2 \Rightarrow 29q^2 = p^2$$

- This says that 29 can divides a^2 which means 29 divides a. ~~since~~ since 29 is prime $\Rightarrow 29n = a$
- ~~therefore~~
- $29q^2 = (29n)^2$ where n is some integer
  - this implies that 29 divides q.

This is a contradiction since

③ $\frac{p}{q}$ represent $\sqrt{29}$ in lowest terms.

③ IDK

④ (i)

$A(0) = 0(-1)(-2) = 0$

$A(1) = 0 + 1(0)(-1) = 0$ $\Rightarrow$ 4 cases

$A(2) = 0 + 2(1)(0) = 0$

$A(3) = 0 + 3(2)(1) = 6$

(ii)

Not enough time

⑧

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

$\Rightarrow$

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                  1
                2 x 8
              3 x 9 x 15
            4 x 10 x 16 x 22
          5 x 11 x 17 x 23 x 29
        6 x 12 x 18 x 24 x 30 x 36
      7 x 13 x 19 x 25 x 31 x 37 x 43
        14 x 20 x 26 x 32 x 38 x 44
          21 x 27 x 33 x 39 x 45
            28 x 34 x 40 x 46
              35 x 41 x 47
                42 x 48
                  49
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$\Downarrow$

FINAL ANSWER:

4	35	10	41	16	47	22
29	11	42	17	42	23	5
12	36	43	49	24	6	30
37	19	43	25	7	31	13
20	44	26	1	32	14	38
45	27	2	33	8	39	21
28	3	34	9	40	15	46

⑨ "Elements" was the most reproduced & studied, ~~and~~ and the author is Euclid.

⑩ $(V + F - E = 2)$ discovered by Leonard Euler

⑪ $\lim_{n\to\infty}\left(\frac{1}{1}+\frac{1}{2}+\dots+\frac{1}{n}-\log n\right) \approx 0.577216\dots$

$\Rightarrow$ this is Euler's constant

⑫

⑥

Thales (6th Century BC)
Euclid (4th Century BC)
Archimedes (3rd Century BC)
Brahmagupta (6th Century)
Fibonacci (12th Century)
Galileo (14th Century)
Newton (15th Century)
Euler (18th Century)
Gauss (18th Century)
Doron Zeilberger (20th century)

⑦ An egyption fraction is a ~~fraction~~ sum of fractions expressed in the form of $\frac{1}{n}$ where n is any natural number. So any fraction can be rewritten in the form of

$$\frac{a}{b} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \dots + \frac{1}{n_k} \quad \text{(finite)}.$$

$$\frac{5}{6} = x \qquad \cancel{\text{exp}}\left(\frac{6}{5}\right) = \frac{1}{x} \qquad \text{ceiling}\left(\frac{6}{5}\right) = n = 2$$

$$\frac{1}{2} + \left(\frac{5}{6} - \frac{1}{2}\right) \Rightarrow \boxed{\frac{1}{2} + \frac{1}{3}}$$

⑧ Ionian (Greek) math was more conceptual and based in philosophy. It described nature, geometry, and worked in the abstract, focusing less on actual numbers. The father of Greek math is Thales. The Greeks believed that it was used to find "order in chaos and arrange ideas in logical chains." Babylonian math and Chinese math was more technical and numeric. Babylonians used clay tables for alegbraic systems. Chinese math also looked at alegebra and number theory.

42 x 45

49

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(10) $(V + F - E = 2)$ discovered by Leonard Euler

(11) $\lim_{n \to \infty} \left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n} - \log n \right) \approx 0.577216\dots$

$\Rightarrow$ this is Euler's constant

(12)