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Exam 1

1. Assume that there is a finite number of primes and let P be all those primes such that $P = P_1 \cdot P_2 \cdot \ldots P_n$ then we look at the next number lets say $X = P_1 \cdot P_2 \cdot \ldots P_n + 1$. If X is prime then we found a new prime and that contradicts that P is the list of all primes. If X is composite then it can be divisble by $P_k \in \{P_1, P_2, \ldots, P_n\}$. Since P_k can be factored out from P and X then it can be factored out from $P-X$ so $\frac{P-X}{P_k}$ is a integer but

$\frac{P-X}{P_k} = \frac{1}{P_k}$ Since $P-X=1$ and

$\frac{1}{P_k}$ is not an integer thus the contradition

Therefore there are infinite number of primes.

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2. Assume that $\sqrt{29}$ is rational Such that $\sqrt{29} = \frac{p}{q}$ where $q \neq 0$ and

p and q both do not have factors of 29. Looking at $\sqrt{29} = \frac{p}{q}$ then

$29 = \frac{p^2}{q^2}$ so $29q^2 = p^2$ or that

$q^2 = \frac{p^2}{29}$. Since p^2 is divisble by

29 then p is divisible by 29. So $p = 29 \cdot K$ for $K \in \mathbb{Z}$. Now we have $q^2 = \frac{(29 \cdot K)^2}{29} \rightarrow q^2 = 29 \cdot K$ and

by the Same argument q is divisble by 29. but our assumption Says p and q both do not have factors of 29 and we found they both do so thats the contradiction. Therefore $\sqrt{29}$ is irrational.

3. $\arctan x = \int_0^x \frac{1}{1+t^2}\,dx = \int_0^x \underbrace{\frac{1}{1-(-t^2)}}_{\text{geometric series}}\,dx$

$$= \int_0^x \sum_{n=0}^{\infty} (-t^2)^n$$

$$= \int_0^x \sum_{n=0}^{\infty} (-1)^n t^{2n}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = \arctan x$$

Now $\arctan(x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n+1}}{2n+1}$

By just substituting in x^3 for x

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4. $\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$

I) it is degree 4 so you would have to verify it is true for 5 different cases.

$n=0$

$\sum_{k=0}^{0} k(k-1)(k-2) = 0(0-1)(0-2) = 0$

$\frac{(0+1)(0)(0-1)(0-2)}{4} = 0$ ✓

$n=1$

$\sum_{k=0}^{1} k(k-1)(k-2) = 0 + 1(1-1)(1-2) = 0$

$\frac{(1+1)1(1-1)(n-2)}{4} = 0$ ✓

$n=2$

$\sum_{k=0}^{2} k(k-1)(k-2) = 0+0+2(2-1)(2-2) = 0$

$\frac{(2+1)(2)(2-1)(2-2)}{4} = 0$ ✓

$n=3$

$\sum_{k=0}^{3} k(k-1)(k-2) = 0+0+0+3(3-1)(3-2) = 6$

$\frac{(3+1)(3)(3-1)(3-2)}{4} = \frac{(4)(3)(2)(1)}{4} = 6$ ✓

$n=4$

$\sum_{k=0}^{4} k(k-1)(k-2) = 0+0+0+6+4(4-1)(4-2) = 6+4(3)(2) = 30$

$\frac{(4+1)(4)(4-1)(4-2)}{4} = (5)(3)(2) = 30$ ✓

Since they are both degree 4, and agree up to 5 different values then

$\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$

is true

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II) Base case $n=0$, $\sum_{k=0}^{0} k(k-1)(k-2) = 0$

$\frac{(0+1)(0)(0-1)(0-2)}{4} = 0$ and $0=0$

thus it is true for $n=0$.

Now Assume $\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$

for some $n \in \mathbb{N}$. we need to prove

$\sum_{k=0}^{n+1} k(k-1)(k-2) = \frac{((n+1)+1)(n+1)((n+1)-1)((n+1)-2)}{4}$

So $\sum_{k=0}^{n+1} k(k-1)(k-2) = \sum_{k=0}^{n} k(k-1)(k-2) + (n+1)(n+1-1)(n+1-2)$

$= \frac{(n+1)n(n-1)(n-2)}{4} + (n+1)(n)(n-1)$ By our assumption.

$= \frac{(n+1)n(n-1)(n-2)+4(n+1)(n)(n-1)}{4}$

and if we do some algebra we see that

$\sum_{k=0}^{\infty} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$

for all $n \in \mathbb{N}$

5

1

2 8

3 9 15

4 10 16 22

5 11 17 23 29

6 12 18 24 30 36

7 13 19 25 31 37 43

14 20 26 32 38 44

21 27 33 39 45

28 34 40 46

35 41 47

42 48

49

7x7

4	35	10	41	16	47	22
29	11	42	17	48	23	5
12	36	18	49	24	6	30
43	19	37	25	13	31	7
20	44	26	1	32	14	38
45	27	2	33	8	39	21
46	15	40	9	34	3	28

6. Newton - 17th Century
Archimedes - (287 - 212)
Galileo - 16th century
Euler - 18th century
Zeilberger - 21^{st}
Euclid - 323 BC
Thales - 624 BC
Brahmagupta - 625
fibinacci - 12th

Thales, Euclid, Archimedes, Brahmagupta, fiblnaci, Galilieo, Newton, Euler, Zeiberger

7. Egyptian fraction is a sum of fractions with 1 as the numerator for each of the summands.

$$\frac{5}{6} = \frac{1}{2} + \frac{5}{6} - \frac{1}{2} = \frac{1}{2} + \frac{2}{6} = \frac{1}{2} + \frac{1}{3}$$

8. Ionian tried to study the science and arts of older civilizations and not reproduce them. Babylonian math were more devolped than its oriental counterparts. Chinese math was just general arthimetic algebraic nature. Euclid is the father of greek math

9. The "Elements" made by Euclid.

10. $V+F-E=2$ made by Euler

11. $\lim_{n \to \infty} \left(\frac{1}{1} + \frac{1}{2} + \ldots + \frac{1}{n} - \log n \right) = .577216\ldots$

which is the constant of Euler.

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12. $\sin(\sin x) = \sin x - \frac{1}{6}(\sin x)^3$

$= x - \frac{1}{6}x^3 + \dots - \frac{1}{6}(x - \frac{1}{6}x^3 + \dots)^3$

$\sin(\sin(\sin x)) = (x - \frac{1}{6}x^3 + \dots - \frac{1}{6}(x - \frac{1}{6}x^3 + \dots)^3)$

$- \frac{1}{6}(x - \frac{1}{6}x^3 + \dots - \frac{1}{6}(x - \frac{1}{6}x^3 + \dots)^3)^3$

Just have to worry about up to x^3

$a_0 = 0,\ a_1 = 1,\ a_2 = 0,\ a_3 = -\frac{1}{3}$

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