NAME: (print!) Kishan Patel

Email DrZlinear@gmail.com as soon as I tell you (around 4:20 pm)
Subject: mt1
with an attachment called
mt1FirstLast.pdf (e.g. mt1LehonardEuler.pdf)
E-Mail address: kpp89@scarletmail.rutgers.edu

MATH 437 Exam I for Dr. Z.'s, Fall 2021 (Oct. 27, 2021)

No Calculators! No Cheatsheets! YOU MAY USE YOUR HISTORY NOTEBOOK (But not your Math Notebook)
Show your work! An answer without showing your work will get you zero points.
A random subset of the students will be picked for short (private!) chats via WebEx, in order to verify that there was no outside help

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

11. (out of 10)

12. (out of 10)

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total: (out of 120)

1. (10 pts.) Prove that there are infinitely many primes.

Suppose n is the finite amount of primes.

$P = P_1 \cdot P_2 \cdot P_3 \cdot \ldots P_n + 1$

$P >$ than all other primes. and isn't prime.

this P must be divided by a prime number, however there will be a remainder of 1, this making P prime causing a contradiction

2. (10 pts.) Prove that $\sqrt{29}$ is irrational.

$$(\sqrt{29})^2 = \left(\frac{a}{b}\right)^2$$

$$29 = \frac{a^2}{b^2}$$

$$29b^2 \neq a^2$$

3. (10 pts) Derive (from scracth, only using geometric series, and calculus) the Taylor series around $x = 0$ of the function

$$\arctan x^3 \quad .$$

Explain all steps!

4. (10 pts. altogether) Prove that

$$\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4} \quad ,$$

(i) (5 points): The Dr. Z. way (verifying it for sufficiently many special cases, explain how many you need)

$K=0;\ 0(0-1)(0-2)=0$; $n=0;\ \frac{(0+1)\cdot 0\cdot(0-1)(0-2)}{4}=0$

$K=1;\ 1(1-1)(1-2)=0+0$; $n=1;\ \frac{(1+1)\cdot 1\cdot(1-1)(1-2)}{4}=0$

$K=2;\ 2(2-1)(2-2)=0+0+0$; $n=2;\ \frac{(2+1)\cdot 2\cdot(2-1)(2-2)}{4}=0$

$K=3;\ 3(3-1)(3-2)=6$; $n=3;\ \frac{(3+1)\cdot 3(3-1)(3-2)}{4}=6$

$K=4;\ 4(4-1)(4-2)=24+6=30$; $n=4;\ \frac{(4+1)\cdot 4\cdot(4-1)(4-2)}{4}=30$

need at least 5 cases to find zeros, then the divisor.

(ii) (5 points): The traditional way, using **complete mathematical induction**.

$$\sum_{K=0}^{n-1} K(K-1)(K-2) = \frac{n(n-1)(n-2)(n-3)}{4}$$

$$\sum_{K=0}^{n} K(K-1)(K-2) = \left(\sum_{K=0}^{n-1} K(K-1)(K-2)\right) + n(n-1)(n-2)$$

$$= \frac{n(n-1)(n-2)(n-3)}{4} + n(n-1)(n-2)$$

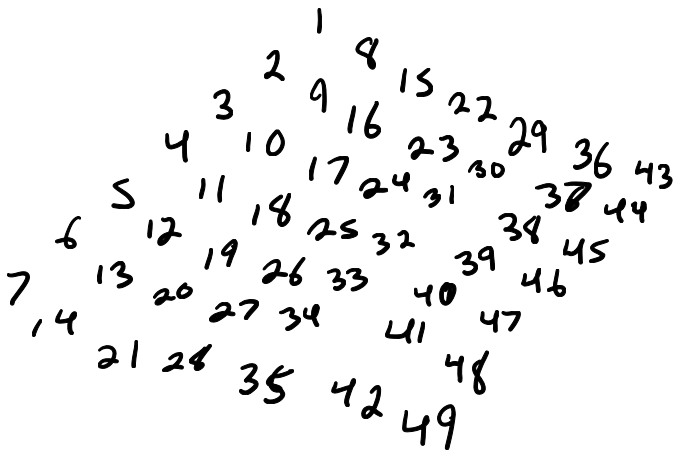
$$= \frac{n(n-1)(n-2)(n-3)}{4} + \frac{4n(n-1)(n-2)}{4}$$

$$= \frac{n[(n-1)(n-2)(n-3) + 4(n-1)(n-2)]}{4}$$

$$= \frac{n[n^3 - 6n^2 + 11n - 6 + 4(n^2 - 3n + 2)]}{4}$$

$$= \frac{n[n^3 - 2n^2 - n + 2]}{4} = \frac{n(n-1)(n-2)(n+1)}{4} \quad \surd$$

5. (10 points) Construct a seven by seven Magic Square.



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6. (10 points) Arrange the following people according to their year-of-birth, from oldest to youngest.

Newton, Archimedes, Gallileo, Euler, Gauss, Zeilberger, Euclid, Thales, Brahmagupta, Fibonacci.

For each person, state their century of birth.

Thales, Euclid, Archimedes, Brahmagupta, Fibonacci, Gallileo, Newton, Gauss, Euler, Zeilberger

7. (10 points). What is an Egyptian fraction? Express $\frac{5}{6}$ as an Egyptian fraction

An egyption fraction is a fraction made of sums with the numerator of 1

$\frac{5}{6} = \frac{1}{2} + Egf(\frac{2}{6})$

$= \frac{1}{2} + \frac{1}{3}$

8. (10 points) What is the difference between Ionian (Greek) mathematics and ancient Babylonian and Chinese mathematics? Who was the traditional father of Greek mathematics?

Archimedes was the father of mathematics

Greek mathematics were based on applied & practical math

where babylonian & chinese math is more number based.

9. (10 points) What book, except for the bible, was the most reproduced and studied in the Western world? Who was its author?

Elements, by Euclid

10. (10 points) In a closed polyhedron, what is a relation between V, the number of vertices, E, the number of edges, and F, the number of faces? Who is it due to?

Euler discovered the relation of

$V - E + F = 2$

11. (10 points) What is the name of the following constant:

$$\lim_{n\to\infty}\left(\frac{1}{1}+\frac{1}{2}+\ldots+\frac{1}{n}-\log n\right) \quad .$$

What is its approximate value? Euler's constant

0.577

12. (10 points) Using the beginning of the famous Taylor expansion, about $x = 0$ for $\sin x$, namely

$$\sin(x) = x - \frac{1}{6}\,x^3 + \ldots \quad ,$$

find the beginning (up to term x^3) of the Taylor series, about $x = 0$ of

$$f(x) = \sin\sin\sin x \quad ,$$

in the form

$$a_0 + a_1 x + a_2 x^2 + a_3 x_3 + \ldots$$

Ans.:

$$a_0 = \quad ; \quad a_1 = \quad ; \quad a_2 = \quad ; \quad a_3 = \quad .$$

$f(x) = \sin(\sin(\sin(x)))$ $f(0) = 0$

$f'(x) = \sin\sin(\cos(x)) + \sin\cos(\sin x)$

$f(x) \approx \frac{f(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$