History of Moth | Exom 1 | Koron Amin

① Assume there are finitely mony primes $p_1, p_2, \ldots, p_k$

Then let $N = p_1 p_2 \cdots p_k + 1$. Either N is prime or not. If it is prime then we hove a controdiction since we hove a new prime N not on the list $p_1, p_2, \ldots, p_k$. If it isn't prime then N would divide $p_1 p_2 \cdots p_k - 1$, which is impossible, hence N is onother prime not on the list. Meoning $p_1 p_2 \cdots p_k$ ore not oll the primes. Hence primes ore infinite.

② Assume $\sqrt{29}$ is irrotionol. Then we con write $\sqrt{29} = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$ in lowest terms. Then $29 = \frac{a^2}{b^2} \rightarrow 29b^2 = a^2$

Therefor 29 divides a^2, but since 29 is prime, 29 divides a, therefor $29k = a$, hence $29b^2 = (29k)^2 = 29^2 k^2$

which meons $b^2 = 29k^2$ hence 29 divides b^2 ond 29 divides b. This is o controdiction since we soid $\frac{a}{b}$ ore in lowest terms but we con divide both a and b by 29, hence $\sqrt{29}$ is irrotionol.

■

3) We know that

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

then

$$\arctan(x^3) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n+1}}{2n+1} = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{(x)^{6n+3}}{2n+1}}$$

4) Prove
(i)

$$\sum_{k=0}^{n} k(k-1)(k-2) = \frac{n(n+1)(n-1)(n-2)}{4}$$

Let $A(n) = \frac{n(n+1)(n-1)(n-2)}{4}$ → We know $A(n)$ is a polynomial of degree 4

Hence we ~~also~~ need to show for $\underline{A(0), A(1), A(2), A(3)}$ 4 cases

$A(0)$ $n=0$: $0(-1)(-2) = 0 = \frac{0(1)(-1)(-2)}{4} = 0$ ✓

$A(1)$ $n=1$: LHS $0 + 1(0)(-1) = 0$ | RHS $\frac{1(2)(0)(-1)}{4} = 0$ ✓

$A(2)$ $n=2$: LHS $0 + 2(1)(0) = 0$ | RHS $\frac{2(3)(1)(0)}{4} = 0$ ✓

$A(3)$ $n=3$: LHS $0 + 3(2)(1) = 6$ | RHS $\frac{3(4)(2)(1)}{4} = \frac{24}{4} = 6$ ✓

Since both sides are poly of degree 3, and agree of 4 different values, they are the same ▣

4 (ii)

$$\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$$

Base $n=0$, LHS: $0(-1)(-2) = 0$ and the ~~RHS~~ RHS: $\frac{1(0)(-1)(-2)}{4} = 0$

Hence the equation is true for $n=0$.

Assume its true for ~~for~~ some $m \in \mathbb{N}$, hence

$$\sum_{k=0}^{m} k(k-1)(k-2) = \frac{(m+1)m(m-1)(m-2)}{4}$$

Then for $m+1$ we have

$$\sum_{k=0}^{m+1} k(k-1)(k-2) = \left[\sum_{k=0}^{m} k(k-1)(k-2)\right] + \left((m+1)(m)(m-1)\right)$$

$$= \frac{(m+1)m(m-1)(m-2)}{4} + \frac{4(m+1)m(m-1)}{4}$$

$$= \frac{(m+1)m(m-1)(m-2) + 4(m+1)m(m-1)}{4}$$

$$= \frac{(m+1)m(m-1)\left[(m-2)+4\right]}{4} = \frac{(m+1)m(m-1)(m+2)}{4}$$

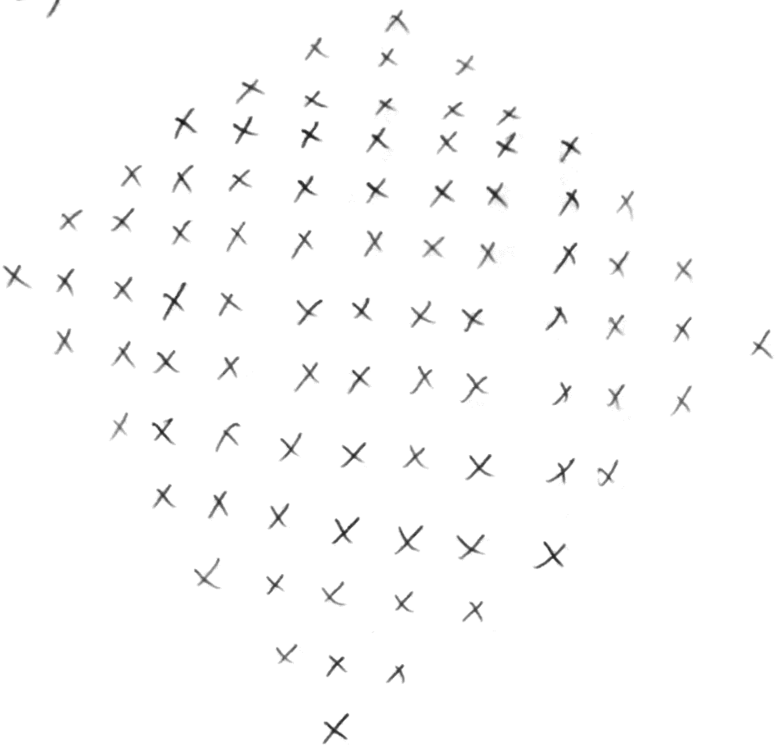
Hence since $\sum_{k=0}^{m+1} k(k-1)(k-2) = \frac{(m+1)m(m-1)(m+2)}{4}$

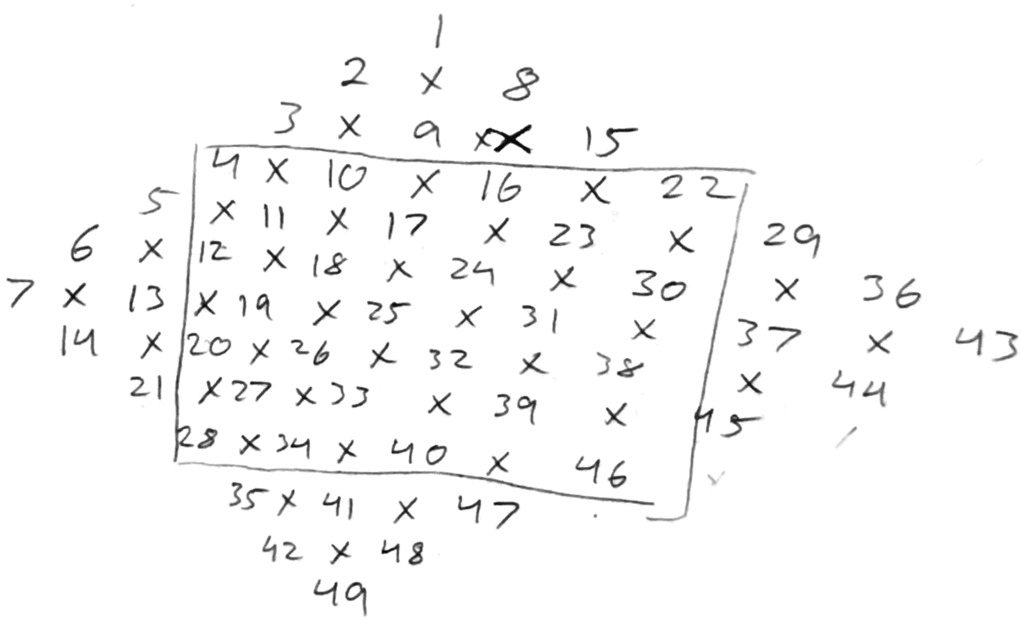
by mathematical induction, $\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$

for all $n \in \mathbb{N}$.

$\blacksquare$

5)




1
2 x 8
3 x 9 x 15
4 x 10 x 16 x 22
5 x 11 x 17 x 23 x 29
6 x 12 x 18 x 24 x 30 x 36
7 x 13 x 19 x 25 x 31 x 37 x 43
14 x 20 x 26 x 32 x 38 x 44
21 x 27 x 33 x 39 x 45
28 x 34 x 40 x 46
35 x 41 x 47
42 x 48
49

*

4	35	10	41	16	47	22
29	11	42	17	48	23	5
12	36	18	49	24	6	30
37	19	43	25	7	31	13
20	44	26	1	32	14	38
45	27	2	33	8	39	21
28	3	34	9	40	15	46

6) Newton - [1643, 1727] - 17th century ✓

* Archimedes - [287 BC, 212 BC] ✓ 3rd century BC

Gallileo - [1564, 1642] - 16th century ✓

Euler - [1707, 1783] - 18th century ✓

Gauss - [1777, 1855] - 18th century ✓

Zeilberger - [1950, current] - 20th century ✓

* Euclid - [325 BC, 265 BC] ✓ 4th century BC

* Thales - [626-623 BC, 548-545 BC] ✓ 6th century BC

Brahmagupta - [590 AD, 668 AD] - 6th century ✓

Fibonacci - [1170, 1250] 12th century ✓

<u>Order</u>

Thales, Euclid, Archimedes, Brahmagupta, Fibonacci, Gallileo, Newton, Euler, Gauss, Zeilberger

7) Egyption froction is an expression which a serres of froctions who numerotor is 1 and denominotor is a positive integer, all unique.

$$\frac{5}{6} = \frac{1}{2} + \left(\frac{5}{6} - \frac{1}{2}\right) = \frac{1}{2} + \left(\frac{5-3}{6}\right) = \frac{1}{2} + \frac{1}{3}$$

Hence $\frac{5}{6}$ as an EF is $\frac{1}{2} + \frac{1}{3}$

8) Ionion moth wos very conceptual ond was primorly bosed in philosophy. Topics included geometry of noture, ond wos very obstroct. The fother of Greek moth is Thales.

9) "Elements" and its outhor is Euclid

10) $(V + F - E = 2)$ and it wos due to Euler

11) Euler's constont

12) Taylor expansion about $x=0$ for $\sin x$

$$\sin(x) = x - \frac{1}{6}x^3 + \dots$$

Maclurin series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$f(x) = \sin(\sin(\sin(x)))$

$f(0) = 0$

$f'(x) = \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))$

$f'(0) = 1$

$f''(x) = -\sin(x)\cos(\sin(x))\cos(\sin(\sin(x)))$
$- \sin(\sin(x))\cos^2(x)\cos(\sin(\sin(x)))$
$- \sin(\sin(\sin(x)))\cos^2(\sin(x))$

$f''(0) = 0$

Then

$\sin(\sin(\sin(x)))$

$= 0 + \frac{1}{1!}x + \frac{0}{2!}x^2$

$= x +$

$a_0 = 0$

$a_1 = 1$

$a_2 = 0$