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Exam I

1. Assume there is a finite number of prime number:

$p_1, p_2, p_3, \cdots, p_n$

Now let $N = (p_1 \times p_2 \times p_3 \cdots \times p_n) + 1$

Notice that N cannot be devide from any prime number we list. which me

N is a prime number

Here is a contradiction to the assumption, thus, there is an infinite number

prime.

2. Prove that $\sqrt{29}$ is irrational

Suppose there exist a pair of integers $m > n > 0$ such that and they both $GCD(m, n) = 1$

$\sqrt{29} = m/n$ (square both side)

$29 = m^2/n^2$

$m^2 = 29n^2$

Since 29 is a prime number, If m^2 could divide 29, then m can also divide

29 is also a factor of m.

Thus, $m = 29a$

$(29a)^2/29 = n^2$

$29a^2 = n^2$

$a^2 = n^2/29$

$\Rightarrow$ 29 is also a factor of n^2 and n. Therefore, by contradiction 29 is a

common factor of m and n, which is contradictory to our initial assumption.

Thus, square root 29 is irrational

3. $S_n = \frac{a(r^n - 1)}{r - 1}$

Taylor: $\arctan x^3 = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$

$= 0 + \frac{0}{1!}x + \frac{0}{2!}x^2 + \frac{6}{3!}x^3 + \frac{0}{4!}x^4 + \frac{0}{5!}x^5 + \cdots$

$= 0 + 0/_{1!}x + 0/_{3!}x^2 + 6/_{3!}x^3 + 0/_{4!}x^4 + \cdots$

$= x^3 - 1/_3 x^9 \cdots$

4.(i) $n=0$: $0 = 0\cdot(1)(-1)(-2)/4 = 0$

$n=1$: $0 = 0 \; (1)\cdot(0)(-1)/4 = 0$

$n=2$: $0 = 0 \; (1)\cdot(2)\cdot(1)\cdot(0)/4 = 0$

$n=3$: $6 = 6 \; 3\cdot 2\cdot 1 = 4\cdot 3\cdot 2\cdot 1/4 = 6$

$n=4$: $(4\times3\times2)+(3\times2\times1) = 5\cdot4\cdot3\cdot2/4 = 30$

$n=5$ $(5\times4\times3)+(4\times3\times2)+(3\times2\times1) = 90 = 6\times5\times4\times3$

Since both side are polynomial of degree three and they are agree at four different value

(ii) Base $n=1 \Rightarrow 1\cdot(0)\cdot(-1) = (2)\cdot(1)\cdot(-1)\cdot(0) = 0$

Assume $P(n)$ is true is true which is that

$$\sum_{n=0}^{n} n(n-1)(n-2) = (n+1)n(n-1)(n-2)/4$$

For $P(n+1)$ $\quad P(n+1) - P(n) =$

Left hand side $(n+1)n(n-1)$

Right hand side $= (n+2)\cdot(n+1)\cdot(n)(n-1)/4$

$(n+2)\cdot(n+1)\cdot(n)(n-1)/4 - (n+1)n(n-1)(n-2)/4$

$\frac{(n+1)(n)(n-1)}{4}\cdot((n+2)-(n-2))$

$= (n+1)\cdot(n)(n-1)$

Left = Right

Thus, when $P(n)$ is true, $P(n+1)$ is also true. Therefore.

$$\sum_{k=0}^{n} k(k-1)(k-2) = (n+1)n(n-1)(n-2)/4$$

5. 7×7

4	35	10	49	16	47	22
29	11	42	17	48	23	5
12	36	18	41	24	6	30
37	19	43	25	7	31	13
20	44	26	9	32	14	38
45	27	2	33	8	39	21
28	3	34	1	40	15	46

6. Archimedes (c. 287), Euclid (325 bc) Brahmagupta (598. bc), Thales (620 Bc), Fibonacci (1170), Gallileo (14 century), Newton (1 Euler (18 century), Gauss (18 century), Zeilberger (20 century)

7. An Egyptian Fraction is a finite sum of distinct unit fractions such that $1/2 + 1/3 + 1/6$

$5/6 = \frac{1}{2} + \frac{1}{3}$

8 Greek mathematics is based on Greek language and use geometry to problem, babylonian mathematics is a range of numeric mathematic. Chi mathematic focus on number calculating and geometry.

Pythagoras known as the father of Greek mathematics

9.

10. $F + V - E = 2$, Euler's Formula

11. Infinite series, 1

12. $a_0 = 0$ $a_1 = \sin(0)$ $a_2 = x$ $a_3 = -\frac{1}{2}x^3$