Edward Chung Math History 10/27/2021

1) Prove that there are infinitely many primes.

- Suppose there is a finite amnt of primes, and that $p_n$ exists st. $p_1 < p_2 < \dots < p_n$, and $p_n$ is the largest prime. Now suppose some $P = p_1 \cdot p_2 \cdot p_3 \cdot \dots \cdot p_n + 1$. It is clear that none of $p_1, p_2, \dots, p_n$ are factors of $P$. However, every integer can be prime factorized, thus $P$ can be divided by a prime $> p_n$, or $P$ itself is prime. Either result is a contradiction. Thus, there must be a prime $> p_n$, and thus there are infinitely many primes. $\square$

2) Prove that $\sqrt{29}$ is irrational

- Assume $\sqrt{29}$ is rational. Thus, there exists a $p$ and $q$ st:

$$\frac{p}{q} = \sqrt{29} \Rightarrow \frac{p^2}{q^2} = 29 \Rightarrow p^2 = 29q^2$$

☆ and $p/q$ is in simplest form (no common factors) ☆

- Because 29 is prime, we see that $29 \mid p^2 \Rightarrow 29 \mid p$.
- Thus, there is some $r$ st. $29r = p$.
- Substituting, $(29r)^2 = 29q^2 \Rightarrow 29r^2 = q^2$
- Again, we see that $29 \mid q^2 \Rightarrow 29 \mid q$.
- Thus, $p$ and $q$ have a common factor, which is a contradiction.

Thus, $\sqrt{29}$ is irrational. $\square$

3) Taylor $\arctan(x^3)$ $x=0$

a) find first four derivatives

$$f(x) = \tan^{-1}(x^3) \qquad f''(x) = \frac{-6x(2x^6 - 1)}{(x^6+1)^2} \qquad f'''(x) = \frac{6(10x^{12} - 25x^6 + 1)}{(x^6+1)^3}$$

$$f'(x) = \frac{3x^2}{x^6+1}$$

b) Taylor Series $x=0$ $f(0) = 0$ $f''(0) = 0$

$f'(0) = 0$ $f'''(0) = 6$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}a}{n!}(x-a)^n = \frac{f^0(0)}{0!}(x-0)^0 + \frac{f'(0)}{1!}(x-0)^1 + \frac{f^2(0)}{2!}(x-0)^2 + \frac{f^3(0)}{3!}(x-0)^3 + \dots$$

$$= \frac{0(x-0)^0}{1} + \frac{0(x-0)^1}{1} + \frac{0(x-0)^2}{2!} + \frac{6(x-0)^3}{3!} + \dots \qquad \boxed{\approx f(x) = \sum_{n=0}^{\infty} \frac{(x-0)^{2n+1}}{2n+1}}$$

$0 + 0 + 0 + 1 +$

4) Prove $\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$

i) $n=0$ $\quad 0(0-1)(0-2) = (0)(1)(-1)(-2)/4$ ✓

$n=1$ $\quad 0+0 = (0)(1)(-1)(2)/4$ ✓

$n=2$ $\quad 2(1)(0) = (3)(2)(1)(0)/4$ ✓

$n=3$ $\quad 3(2)(1) = (4)(3)(2)(1)/4 = 6$ ✓

$n=4$ $\quad 6+4(3)(2) = (5)(4)(3)(2)/4 = 30$ ✓

True

ii) Base $n=0$

$0(0-1)(0-2) = 0 \Rightarrow \frac{(0+1)(0)(0-1)(0-2)}{4} = 0$ Base case satisfied.

Hypothesis: Show that if $n=m$ is true, then $n=m+1$ is also.

Inductive Case: for any $k \geq 0$, $P(k)$ holds, show $P(k+1)$ holds

Assume $\sum_{k=0}^{m} k(k-1)(k-2) = \frac{(m+1)m(m-1)(m-2)}{4}$ for some $m \in \mathbb{N}$

Show $m+1$ is true:

$$\underbrace{\sum_{k=0}^{m} k(k-1)(k-2)}_{\Downarrow} + (m+1)(m)(m-1) = \frac{(m+2)(m+1)(m)(m-1)}{4}$$

$$4\left[\frac{(m+1)(m)(m-1)(m-2)}{4} + (m+1)(m)(m-1) = \frac{(m+2)(m+1)(m)(m-1)}{4}\right]$$

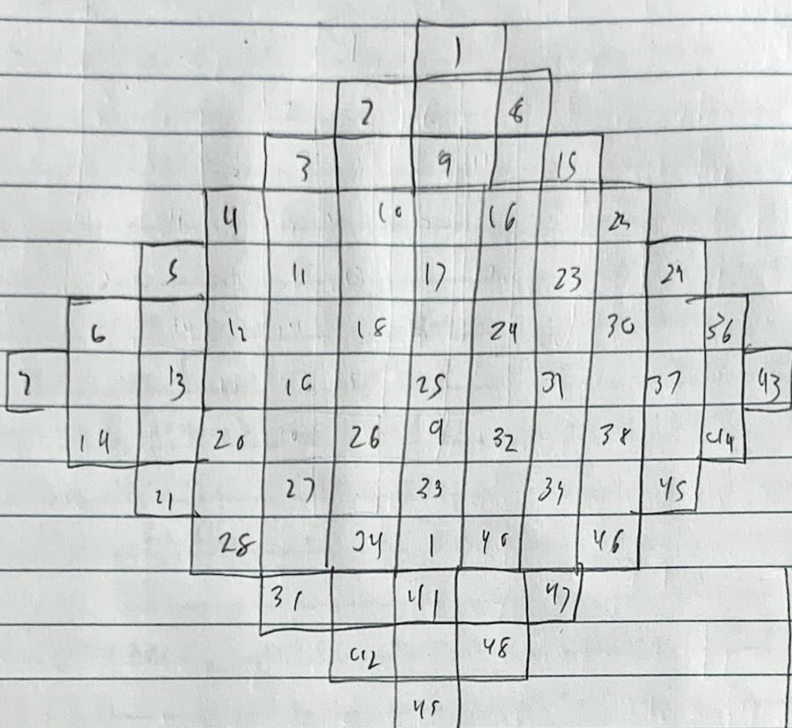
$$\cancel{(m+1)m(m-1)}(m-2) + 4\cancel{(m)(m+1)(m-1)} = (m+2)\cancel{(m+1)(m)(m-1)}$$

$$(m-2)+4 = m+2) \checkmark$$

Both sides are equal

Since the statement is true for $n=0$, and truth of $n=m$ implies $n=m+1$, the statement above is true by complete mathematical induction

5) 7×7 magic square



| | | | | | | |
|---|---|---|---|---|---|---|
| 4 | 42 | 10 | 49 | 16 | 48 | 22 |
| 29 | 11 | 42 | 17 | 48 | 23 | 5 |
| 12 | 36 | 18 | 41 | 24 | 6 | 30 |
| 43 | 19 | 37 | 25 | 13 | 31 | 7 |
| 20 | 49 | 26 | 9 | 32 | 14 | 38 |
| 45 | 27 | 2 | 33 | 8 | 39 | 21 |
| 28 | 2 | 34 | 1 | 40 | 8 | 46 |

6) Thales 654 BC
Euclid ~300 BC
Archimedes 287 BC
Brahmagupta 590 AD
Fibonacci 1170 AD
Gallileo 1564 AD
Newton 1643 AD
Euler 1707 AD
Gauss 1777 AD
Zeilberger 1950 AD

7) Egyptian fractions were how Egyptians had written fractions before hand, with 1 always in the numerator.

$\frac{5}{6} \to \text{ceil}\left(\frac{6}{5}\right) = 2$ $\quad \frac{5}{6} - \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$

ceil (

$\boxed{\text{Thus } = \frac{1}{2} + \frac{1}{3}}$

8) Oriental and Babylonian mathematics were not very proof based, and only asked "How?", and was surface level. Ionian (Greece) mathematics would delve into "Why?" and explain how things came to be.

- The father is Thales,

9) Fibonacci, who wrote 'Liber Abaci'

10) The relation is $\boxed{F + V = E + 2}$, due to Euler.

11) It is the Euler-Mascheroni Constant (or just Euler Constant) Approximately 0.5772 ...

12. $\sin(x) = x - \frac{1}{6}x^3 + \ldots$

$f(x) = \sin(\sin(\sin(x)))$

$a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ldots$

$a_0 = 0 \quad a_1 = 1 \quad a_2 = 0 \quad a_3 = -3$