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10/27

1. Assume a finite number of primes:

$p_1, p_2, p_3, \ldots, p_n$

imagine a number $p = p_1 \cdot p_2 \cdot p_3 \cdots p_n + 1$

- p always has remainder 1 when divided by any given p.

Therefore, p must be prime or divisible by a number $> p_n$

CONTRADICTION : must be infinite p's

2. Assume $\sqrt{29}$ is rational:

$\sqrt{29} = a/b$

$29 = a^2/b^2$

$29b^2 = a^2$

since squared, prime factorizations must have even exponents

Therefore, 29 must exist an odd number of times in b^2. This is CONTRADICTION, since b^2 is squared, and its prime factorization must have an even ~~number of~~ exponent of 29.

Thusly $\sqrt{29}$ must be irrational.

3. $\arctan(x^3)$

$f(x) = \arctan(x^3)$ $\qquad f(0) = 0$

$f'(x) = \frac{3x^2}{x^6+1}$ $\qquad f'(0) = 0$

$f''(x) = \frac{6x - 12x^7}{(x^6+1)^2}$ $\qquad f''(0) = 0$

$f^3(x) = \frac{6(10x^{12} - 25x^6 + 1)}{(x^6+1)^3}$ $\qquad f^3(0) = 6$

...

$= \sum_{n=0}^{\infty} \frac{f^n(a)(x-a)^n}{n!} = 0 + 0 + 0 + \frac{6x^3}{3!} + \dots$

$= x^3 + \dots$

4. i) $\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)n(n-1)(n-2)}{4}$

$n=0 : 0(-1)(-2) = 0 \quad | \quad \frac{(1)0(-1)(-2)}{4} = 0$

$n=1 : 0 + 1(0)(-1) = 0 \quad | \quad \frac{(2)1(0)(-1)}{4} = 0$

$n=2 : 0+0+2(1)(0) = 0 \quad | \quad \frac{(3)2(1)(0)}{4} = 0$

$n=3 : 0+0+0+3(2)(1) = 6 \quad | \quad \frac{(4)3(2)(1)}{4} = 6$ $\quad n=4$ is necessary

~~$n=4 : 6 + 4(3)(2) = 30 \quad | \quad \frac{(5)4(3)(2)}{4} = 30$~~ ~~unnecessary~~

~~$n=5 : 30 + 5(4)(3) = 90 \quad | \quad \frac{(6)5(4)(3)}{4} = 90$~~ $n=5$ unnecessary

QED, since polynomial is of degree 4 and they agree at 5 different values.

ii) Base case: $n=0$ (shown above)

$P(n-1)$ implies $P(n)$

$P(n-1) = \sum_{k=0}^{n-1} k(k-1)(k-2) = \frac{(n)(n-1)(n-2)(n-3)}{4}$

$0(-1)(-2) + 1(0)(-1) + \dots + (n-1)(n-2)(n-3)$

$= \frac{(n)(n-1)(n-2)(n-3)}{4}$

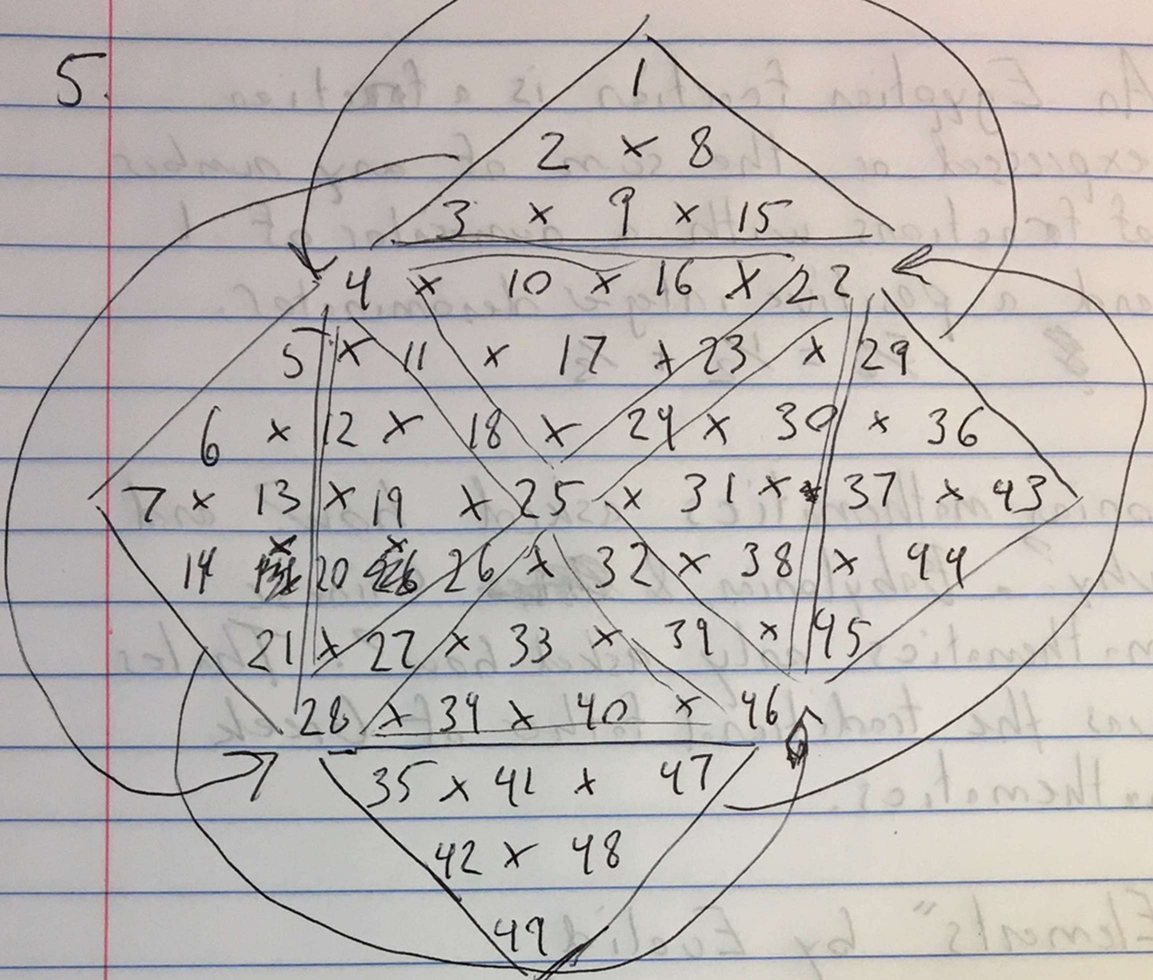
$0 + 0 + \dots + (n-1)(n-2)(n-3) + n(n-1)(n-2)$

$= \frac{(n)(n-1)(n-2)(n-3)}{4} + n(n-1)(n-2)$

$LHS = \frac{(n+1)(n)(n-1)(n-2)}{4}$

TRUE!

5.



4	35	10	41	16	47	22
29	11	42	17	48	23	5
12	36	18	49	24	6	30
37	19	43	25	7	31	13
20	44	26	1	32	14	38
45	27	2	33	8	39	21
28	3	34	9	40	15	46

6. Thales (7^{th} century B.C.), Euclid (4^{th} century B.C.), Archimedes (3^{rd} century B.C.), Brahmagupta (6^{th} century), Fibonacci (12^{th} century), Galileo (16^{th} century), Newton (17^{th} century), Euler (18^{th} century), Gauss (18^{th} century), Zeilberger (20^{th} century)

7. An Egyptian fraction is a fraction expressed as the sum of any number of fractions with a numerator of 1 and a positive integer denominator.

$5/6 = 1/2 + 1/3$

8. Ionian mathematics asked how? and why?. Babylonian & Chinese mathematics only asked how?. Thales was the traditional father of Greek mathematics.

9. "Elements" by Euclid

10. $V + F - E = 2$

This was discovered by Euler

11. This is Euler's constant. It is approximately $\approx .577216\ldots$

12. $f(x) = \sin(\sin(\sin(x)))$ $\quad f(0) = 0$

$f'(x) = \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))$ $\quad f'(0) = 1$

$f''(x) = \ldots$ out of time!