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7. An Egyptian fraction is a representation of a fraction as a sum of fractions with numerator 1. $\frac{5}{6}$, for example, can be represented as $\frac{1}{2} + \frac{1}{3}$

9. Euclid's Elements

6. Thales (7th century BC), Euclid (4th century BC), Archimedes (3rd century BC), Brahmagupta (6th century AD), Fibonacci (12th century AD), Galileo (16th century AD), Newton (17th century AD), Euler (18th century AD), Gauss (18th century AD), Zeilberger (20th century AD)

11. Euler's constant is the nome. Approximate value is 0.577

10. The relation is $V + F - E = 2$ and this is due to Euler

5.

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

8. The father of Greek mathematics is traditionally Thales of Miletus. The difference in the approaches had to do with the separation of arithmetic and logic in Greek math and the different approaches to approximation and computation

2. Assume $\sqrt{29}$ is rational. It must be a ratio of integers p/q . $\sqrt{29} = p/q$. Square both sides to get $29 = p^2/q^2$. Now we have $29q^2 = p^2$. The prime factorizations of q^2 and p^2 will be the squares of the prime factorizations of p and q . This means that we will have a contradiction in that our equation $29q^2 = p^2$ will not have a left side with only prime numbers raised to even exponents.

1. Assume there is a finite list of primes. P is the product of all primes in this list. If $P+1$ is prime we have found a new prime. If not, then $P+1$ is divisible by some other prime p . If this is in our list, then p is the divisor of both P and $P+1$ and subsequently divides the number 1. This is a contradiction.

3. Taylor at $x=0$ of $\arctan(x)$ $\frac{d}{dx} \arctan(x) = \frac{1}{x^2+1} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \quad \arctan(x^3) = f(x)$$

$$\arctan(x^3) = \text{expansion of } (\arctan(x))^3 \text{ at } x=0 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{2n+1}$$

12. $\sin(x) = x - \frac{1}{6}x^3 + \dots$ $f(0) + \frac{f'(0)}{1!} + \frac{f''(0)}{2!} + \frac{f'''(0)}{3!}$

$$\begin{aligned} f(0) &= 0 \\ f'(0) &= 1 \\ f''(0) &= 0 \\ f'''(0) &= -3 \end{aligned}$$

$$\begin{aligned} a_0 &= 0 \\ a_1 &= \frac{1}{1!} = 1 \\ a_2 &= \frac{0}{2!} = 0 \\ a_3 &= \frac{-3}{3!} = -\frac{1}{2} \end{aligned}$$