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Exam 1

1) Assume there are finitely many primes say

$$[P_1, P_2, \dots P_n]$$

we say there exists a # P where;

$$P = P_1 P_2 P_3 \text{———} P_n + 1$$

we see a remainder of 1 when divided and prime #.
Based on the Theorem of Arithmetic,
P must either be Prime or divisible by a prime #
that is larger than P_n. This is a contradition.
Thus, this means there must be infinietly many primes.

2) Suppose $\sqrt{29}$ is rational. Then there exists coprimes 'p' and 'q'
where $q \neq 0$ such that $\sqrt{29} = \frac{p}{q}$

So:
$$\sqrt{29} = \frac{p}{q} \rightarrow 29 = \frac{p^2}{q^2} \rightarrow 29q^2 = p^2$$

with this, 29 must divide p^2. This is only possible
when 29 also divides p.
So there must be an even int n where $p = 29n$
So:
$$29q^2 = p^2 \rightarrow 29q^2 = (29n)^2 \Rightarrow q^2 = 29n^2$$

So 29 must divide q, but we see that it can't as
p and q are coprime. Our assumption is incorrect, and $\sqrt{29}$ is irrational.

arctan x^3

3) we know that $\arctan x = \int_0^x \frac{1}{1+t^2}\,dt$

and the Taylor series

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$$

we can replace z by $-t^2$ to get

$$\frac{1}{1+t^2} = \sum_{n=0}^{\infty} (-t^2)^n = \sum_{n=0}^{\infty} (-1)^n t^{2n}$$

Thus, $\arctan x = \int_0^x \frac{1}{1+t^2} = \int_0^x \sum_{n=0}^{\infty} (-1)^n t^{2n}$

$$= \sum_{n=0}^{\infty} (-1)^n \left.\frac{t^{2n+1}}{2n+1}\right|_0^x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

we plug in x^3 $\quad \sum_{n=0}^{\infty} (-1)^n \frac{(x^3)^{2n+1}}{2n+1}$

4) $\sum_{k=0}^{n} k(k-1)(k-2) = \frac{(n+1)\,n\,(n-1)(n-2)}{4}$

a) $n=0$ $\quad 0 = \frac{(0+1)\,0\,(0-1)(0-2)}{4}$ ✓

$n=1$ $\quad (0)(1)/2 = \frac{(1+1)\,1\,(1-1)(1-2)}{4}$ ✓

$n=2$ $\quad (0)(1)/2 + (1)(2)/2 = \frac{(2+1)(2)(2-1)(2-2)}{4}$ ✓

B) Base Case: $n=0$

$0(0-1)(0-2) = \frac{(0+1)(0)(0-1)(0-2)}{4} \Rightarrow 0=0$ ✓

Inductive hypothesis: $n=m+1$ is true

$$\frac{((n+1)+1)(n+1)((n+1)-1)((n+1)-2)}{4}$$

$$\frac{(n+2)(n+1)(n)(n-1)}{4} + n(n-1)(n-2)$$

5)

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

6) 1) Newton: 1643 → 17th Century 7
2) Archimedes: 3rd Century BCE 1
3) Gallileo: 1564 → 16th Century 6
4) Euler: 1707 → 18th Century 8
5) Gauss: 1777 → 18th Century 9
6) Zeilberger: 1950 → 20th Century 10
7) Euclid: 4th Century 2
8) Thales: 7th Century 4
9) Brahmagupta: 6th Century 3
10) Fibonacci: 12th Century 5

1-10
oldest - youngest

7) $\frac{5}{6}$ $\frac{1}{x} = \frac{6}{5}$ $ceil(\frac{6}{5}) = 2$

$$EF\left(\frac{5}{6}\right) = \frac{1}{2} + EF\left(\frac{5}{6} - \frac{1}{2}\right)$$
$$= EF\left(\frac{1}{3}\right)$$

$$\boxed{\frac{5}{6} = \frac{1}{2} + \frac{1}{3}}$$

8) Ionian: used logic
Babylonian: used algebra, fractions and cubic eqns
Chinese: was concise, decimal system

Archimedes is the father of Greek math

9) Euclids Elements.
The author was Euclid.

10) The relation $(V+F-E=2)$
It is due to Euler.

11) The constant of Euler

The value: 0.577216.....

12) $\sin(x) = x - \frac{1}{6}x^3 + \ldots\ldots$ up to x^3

$f(x) = \sin \sin \sin x$

$f(0) = \sin(\sin(\sin(0)))$

$= x - \frac{1}{3!}x^3$

$f(0) = 0$

$f'(0) = 1$

$a_0 = 0 \quad a_1 = 1 \quad a_2 = 0 \quad a_3 = -\frac{1}{6}$