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Homework 8

(1) Traditional Proof that $\sqrt{2}$ is irrational

<u>Proof</u> - (By contradiction)

Suppose there exists $a, b \in \mathbb{Z}$ s.t. $\sqrt{2} = a/b$.

$$\left(\sqrt{2} = \frac{a}{b}\right)^2 \rightarrow 2 = \frac{a^2}{b^2}$$

$a^2 = 2b^2$. both a & b are positive integer

Then there exists the pair (a,b) with the smallest value of $a+b$.

WLOG (With loss of generality) - The above pair has $a+b$ as small numbers

Intermediate needed fact: the square of an odd integer is always odd

odd number $= 2m+1$

$(2m+1)^2 = \underbrace{(2m)^2 + 4m}_{\text{even}}+1 \rightarrow$ odd integer.

Corrolary - If a^2 is even, then a is even.

Hence, you can write $a = 2n$ for $\exists n \in \mathbb{Z}$.

$(2n)^2 = 2b^2$

$2n = b^2$

Hence b must be an even integer, let us say $b=2m$.

$(2m)^2 = 2n^2$

$4m^2 = 2n^2$

$2m^2 = n^2$

So, (m,n) is also a possibility

$m+n = \frac{(a+b)}{2}$, which is less than (a,b), ※

(2) Geometric proof that $\sqrt{2}$ is irrational

<u>Second Proof (Inspired by Geometric Proof)</u>.- Using Algebra.

$(2b-a)^2 - 2(a-b)^2$

$4b^2 - 4ba + a^2 - 2(a^2 - 2ab + b^2)$

$4b^2 - \cancel{4ba} + a^2 - 2a^2 + \cancel{4ab} - 2b^2$

$2b^2 - a^2$

Algebraic Identity: $(2b-a)^2 - 2(a-b)^2 = -(a^2 - 2b^2)$

Suppose that there exist integers a and b such that $a^2 - 2b^2 = 0$, then

Let (a,b) be the pair with smallest sum $(a=b)$.

Define $c = 2b-a$ $d = a-b$.

$c^2 - 2d = 0$

But $c+d = 2b - a + a - b = b < a+b$ ※.

(3) Convert the following rational numbers into a simple continued fraction

(a) $\frac{29}{16} = 1 + \frac{1}{4 + \frac{1}{3}}$

(b) $\frac{32}{19} = 1 + \frac{1}{2 + \frac{1}{6}}$

(4) $$\sqrt{2} + 1 = 1 + \boxed{\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \cdots}}}}$$

Proves that $\sqrt{2}$ is irrational

(5) $$\frac{(\sqrt{3} + 1)}{2} = 1 + \boxed{\frac{1}{2 - \frac{1}{2 - \frac{1}{\cdots}}}}$$

This is irrational, thus proving the sum would be irrational