NAME: Tianyi He
E-MAIL ADDRESS: th586@scarletmail.rutgers.edu

1. Proof by contradiction.

Suppose $\sqrt{2}$ is rational, and there exists $m > n > 0$, such that

$$\sqrt{2} = \frac{m}{n}, \text{ where } m \text{ and } n \text{ are integers.}$$

And there exists a pair such that it sum, $m+n$ is smallest.

Since $2 = \frac{m^2}{n^2}$, $m^2 = 2n^2$, then m^2 is even, and m is even.

Then $m = 2p$, where p is an integer.

Plugging it in, we have $(2p)^2 = 2n^2$, $4p^2 = 2n^2$, $2p^2 = n^2$,

So we get $n > p > 0$, such that $\sqrt{2} = \frac{n}{p}$, and $n+p = n + \frac{m}{2} < n+m$,

Contradicting with $m > n > 0$ was the pair with the smallest sum.

2. Suppose $\sqrt{2}$ is rational. Then $\sqrt{2} = \frac{a}{b}$ where a, b are integers and b is not zero.

Then $a^2 = 2b^2$, and we have $a^2 - 2b^2 = -((2b-a)^2 - 2(a-b)^2)$.

Suppose there exists positive integer a, b such that $a^2 - 2b^2 = 0$,

then there exists a pair for which $a+b$ is the smallest,

then suppose $c = 2b - a$, $d = a - b$. We have $c^2 - 2d^2 = 0$.

But $c + d = 2d - a + a - b = b < a + b$.

Contradicting that (a, b) was the smallest pair.

3. (a) $$\frac{29}{16} = 1 + \frac{13}{16} = 1 + \frac{1}{\frac{16}{13}} = 1 + \frac{1}{1 + \frac{3}{13}} = 1 + \frac{1}{1 + \frac{1}{\frac{13}{3}}}$$

$$= 1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{3}}}$$

$[1, 1, 4, 3]$ for short.

(b) $\dfrac{32}{19} = 1+\dfrac{13}{19} = 1+\dfrac{1}{\frac{19}{13}} = 1+\dfrac{1}{1+\frac{6}{13}} = 1+\dfrac{1}{1+\frac{1}{\frac{13}{6}}}$

$$= 1+\cfrac{1}{1+\cfrac{1}{2+\frac{1}{6}}}$$

[1, 1, 2, 6] for short.

4. $\sqrt{2}+1 = 2+\sqrt{2}-1$, $\dfrac{(\sqrt{2}-1)(\sqrt{2}+1)}{\sqrt{2}+1} = \dfrac{1}{1+\sqrt{2}}$

$$\sqrt{2}+1 = 2+\frac{1}{1+\sqrt{2}} = 2+\frac{1}{1+(1+\frac{1}{1+\sqrt{2}})} = 2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cdots}}}}$$

So $\sqrt{2}$ is irrational.

5. $\dfrac{\sqrt{3}+1}{2} = 1+\left(\dfrac{\sqrt{3}+1}{2}-1\right)$

$$\frac{\sqrt{3}+1-2}{2} = \frac{\sqrt{3}-1}{2} = \frac{(\sqrt{3}-1)(\sqrt{3}+1)}{2(\sqrt{3}+1)} = \frac{2}{2(\sqrt{3}+1)} = \frac{1}{1+\sqrt{3}}$$

$$\frac{\sqrt{3}+1}{2} = 1+\frac{1}{1+\sqrt{3}} = 1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cdots}}}}}}$$

So $(\sqrt{3}+1)/2$ is irrational.