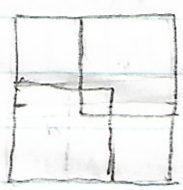
Sarah Catoric Homework 8 10/10/2021

1. Assume to the contrary that $\sqrt{2}$ is rational, so it can be written in the form $\frac{m}{n} = \sqrt{2}$, with m, n integers and $\frac{m}{n}$ in simplest form. Squaring both sides, we get that $\frac{m^2}{n^2} = 2$. Multiplying both sides by n^2, we get $m^2 = 2n^2$. Since $2 \mid m^2$, then m^2 must be even. Therefore, m must be even. So m can be written in the form $= 2a$, where a is an integer. Substituting that into the original equation, we get that $(2a)^2 = 2n^2 \Rightarrow$ $4a^2 = 2n^2$. Dividing both sides by 2 yields $n^2 = 2a^2$. Since $2 \mid n^2$, then n^2 must be even. So, therefore, n must be even. This would mean that $\frac{m}{n}$ would not be in simplest form because $2 \mid n$ and $2 \mid m$, but we assumed that in the beginning. Therefore, $\sqrt{2}$ must be irrational.

2. Construct a square with side lengths a, and place inside it two squares with side length b, each of which has area half the area of the square with side length a. Then construct it in this way:



The center square would have area $(2b-a)^2$

3. a) $\frac{29}{16} = 1 + \frac{13}{16} = 1 + \frac{1}{16/13} = 1 + \frac{1}{1+\frac{3}{13}} = 1 + \frac{1}{1+\frac{1}{13/3}} = 1 + \cfrac{1}{1+\cfrac{1}{4+\frac{1}{3}}}$

so the solution is $[1, 1, 4, 3]$

b) $\frac{32}{19} = 1 + \frac{13}{19} = 1 + \frac{1}{19/13} = 1 + \frac{1}{1+\frac{6}{13}} = 1 + \frac{1}{1+\frac{1}{13/6}} = 1 + \cfrac{1}{1+\cfrac{1}{2+\frac{1}{6}}}$

so the solution is $[1, 1, 2, 6]$

4. The continued fraction for $\sqrt{2}$ is $[1, 2^{\infty}]$, so $1+\sqrt{2}$ would be $[2, 2^{\infty}]$. This is an irrational number because it is an infinite continued fraction. Since 1 is rational, it must be that $\sqrt{2}$ is irrational.

$1 + \frac{1}{\sqrt{3}}$

5. $\frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1}{\frac{\sqrt{3}}{2} + \frac{1}{2}} = \frac{\frac{1}{2} - \frac{\sqrt{3}}{2}}{\frac{1}{4} - \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{1}{2} - \frac{\sqrt{3}}{2}}{\frac{1}{4} - \frac{3}{4}} = \frac{\frac{1-\sqrt{3}}{2}}{-\frac{1}{2}}$

$= -1 + \sqrt{3}$