Quin Buob ok to Post

HW 8

1) Pf: Prove $\sqrt{2}$ is irrational

Assume that $\sqrt{2}$ is the ratio of 2 positive integers, a and b, s.t.

$$\sqrt{2} = \frac{a}{b}$$

$\sqrt{2} = \frac{a}{b}$ square both sides

$2 = \frac{a^2}{b^2}$ rearrange

$a^2 = 2b^2$

If this exists, then there exists the pare (a,b) w/ the smallest possible value of $(a+b)$ WLOG

We know the square of an odd number is odd and the square of an even number is even:

$(2n+1)^2 = 4n^2+4n+1 = 2\cdot 2(n^2+n)+1$

$m = 2(n^2+n) \Rightarrow 2m+1$ odd number

$(2n)^2 = 4n^2 = 2\cdot 2n^2$ $m = 2n^2 \Rightarrow 2m$ even number

From this: If a^2 is even then a must be even

$a = 2n$ for some integer $n \Rightarrow n = \frac{a}{2}$

$a^2 = 2b^2$

$(2n)^2 = 2b^2$

$4n^2 = 2b^2$

$b^2 = 2n^2$

By the same logic b must be an even integer b/c b^2 is even

$b = 2m$ for som integer $m \Rightarrow m = \frac{b}{2}$

$(2m)^2 = 2n^2$

$4m^2 = 2n^2$

$2m^2 = n^2$

$\sqrt{2} = \frac{n}{m}$

There for (m,n) is also a solution for $\sqrt{2}$

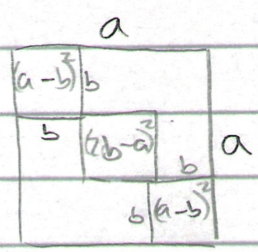
but:

$$(m+n) = \frac{a+b}{2} < (a+b)$$

This is a contradiction b/c $(a+b)$ is the smallest possible solution so $(m+n)$ cannot be smaller than $(a+b)$ $\rightarrow\leftarrow$

QED

2) Prove $\sqrt{2}$ is irrational geometrically



By the law of conservation of area $2(a-b)^2$ must be equal to $(2b-a)^2$

From this:

$(2b-a)^2 - 2(a-b)^2 = 0$

$4b^2 - 4ba + a^2 - 2(a^2 - 2ab + b^2) = 0$

$\cancel{4}^{2}b^2 - \cancel{4ba} + \cancel{a^2} - 2a^2 + \cancel{4ba} - \cancel{2b^2} = 0$

$(a^2 - 2b^2) = 0$

Suppose there exists integers (a,b) s.t. $a^2 - 2b = 0$

Let (a,b) be the pair with the smallest sum $(a+b)$ WLOG

We can define $c = 2b - a$, $d = a - b$

$\Rightarrow c^2 - 2d^2 = 0$

but: $(c+d) = 2b - a + a - b = b < a+b$ $\rightarrow\leftarrow$ so $\sqrt{2}$ cannot be rational

3) a) $\frac{29}{16} = 1 + \frac{13}{16} = 1 + \frac{1}{16/13} = 1 + \frac{1}{1+3/13} = 1 + \frac{1}{1+\frac{1}{13/3}} = 1 + \frac{1}{1+\frac{1}{4+1/3}} =$

$= 1 + \frac{1}{1+\frac{1}{4+1/3}} \Rightarrow [1, 1, 4, 3]$

b) $\frac{32}{19} = 1 + \frac{13}{19} = 1 + \frac{1}{19/13} = 1 + \frac{1}{1+6/13} = 1 + \frac{1}{1+\frac{1}{13/6}} = 1 + \frac{1}{1+\frac{1}{2+\frac{1}{6}}}$

$[1, 1, 2, 6]$

4) $x = 1 + \sqrt{2} = 2 + (\sqrt{2}-1) = 2 + \frac{1}{1/\sqrt{2}-1} = 2 + \frac{1}{2+(\sqrt{2}-1)} = 2 + \frac{1}{2+\frac{1}{2+\dots}}$

$[2, 2, 2, 2 \dots]$

$\sqrt{2}$ must be irrational because 1 is a rational number and a rational number plus an irrational is irrational, and from the continuous fraction we know $\sqrt{2}+1$ must be irrational.

5) $x = 1 + \left(\frac{\sqrt{3}+1}{2} - 1\right) = 1 + \frac{1}{1+\left(\frac{\sqrt{3}+1}{2}-1\right)}$

$$= 1 + \cfrac{1}{2 + \cfrac{1}{\frac{\sqrt{3}+1}{2} - 1}} = 1 + \cfrac{1}{2 + \cfrac{1}{1 + \dots}}$$

$[1, 2, 1, 2, \dots]$

From the ∞ continuous fraction this number must be irrational