MATH 437 HOMEWORK 8 - NINA CHALGERI (Do not post on site)

① $\sqrt{2}$ is irrational.

Proof.

- A rational number can be written in the form $\frac{a}{b}$ where $a, b \in \mathbb{N}$.
- $\frac{a}{b}$ can also be ~~an~~ rewritten as :

$$\frac{a}{b} = q + \frac{r}{b} \quad \text{where} \quad q, r, a, b \in \mathbb{N}$$

- If $r = 0$, then $\frac{a}{b}$ is an integer
- otherwise, $\frac{b}{r} > 1$ so $\frac{b}{r} = s + \frac{t}{r}$ and so on.
- For $\sqrt{2}$,

$$\sqrt{2} = 1 + \frac{(\sqrt{2}-1)}{1}$$

$$\frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{1} = \sqrt{2}+1 = 2 + (\sqrt{2}-1)$$

- Therefore, $\sqrt{2}$ is irrational because it will never end. ~~since we continue~~

② $a^2 - 2b^2 = -((2b-a)^2 - 2(a-b)^2)$

- ~~a, b > 0~~ If $a, b > 0$ such that $a^2 - 2b^2 = 0$, then there must be a pair for which ~~a+b~~ $a+b$ is the smallest
- Let :

$$c = 2b - a \quad \text{and} \quad d = a - b$$

$$\Rightarrow \quad c + d = 2b - a + a - b = b < a + b$$

- (c, d) satisfies the condition that $\sqrt{2} = \frac{c}{d}$ but the sum of c and d is smaller which is a contradiction. (a, b) are not the smallest. Therefore $\sqrt{2}$ is irrational

③ (a) $\frac{29}{16} = 1 + \frac{13}{16} = 1 + \frac{1}{\frac{16}{13}} = 1 + \frac{1}{1 + \frac{3}{13}}$

(b) $\frac{32}{19} = 1 + \frac{13}{19} = 1 + \frac{1}{\frac{19}{13}} = 1 + \frac{1}{1 + \frac{13}{19}}$

④ $\sqrt{2} + 1 = 1 + \frac{\sqrt{2}}{1} = 1 + \frac{1}{1/\sqrt{2}} = 1 + \frac{1}{1 + (\frac{1}{\sqrt{2}} - 1)}$ $\Rightarrow$ stuck in infinite loop $\Rightarrow$ $\sqrt{2}$ is irrational

⑤ $\frac{\sqrt{3}+1}{2} = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1}{2} + \frac{1}{\frac{2}{\sqrt{3}}} = \frac{1}{2} + \frac{1}{1 + (\frac{2}{\sqrt{3}} - 1)}$

$\hookrightarrow$ stuck in loop $\Rightarrow$ irrational