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Hw 8

(1.) $\sqrt{2}$ irrational proof:

To prove this we will use a proof by contradiction.

Let's say for the sake of contradiction, that $\sqrt{2}$ is rational. That means $\sqrt{2} = \frac{m}{n}$, for some $m, n \in \mathbb{Z}$ who are coprime ($\gcd(m,n) = 1$).

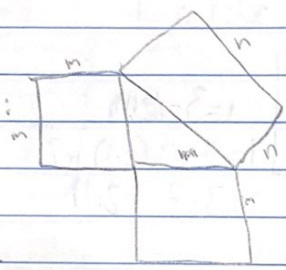
$\sqrt{2} = \frac{m}{n} \Rightarrow 2 = \frac{m^2}{n^2}, \; 2n^2 = m^2$

Since $m^2 = 2n^2$ means that m^2 is even, thus m is even. If m is even, $2n^2$ means that m is divisible by 4, so n^2 must be even and n must also be even.

Since m, n are both even, they share a $\gcd(m,n) > 1$. Therefore, m, n cannot be coprime and $\sqrt{2}$ must be rational!

(2.) Using $a^2 + b^2 = c^2$,

Suppose we have two squares m:

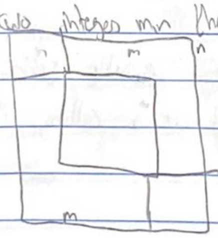


Suppose $m, n \in \mathbb{Z}^+$, then $m^2 + m^2 = n^2$

So we say $2m^2 = n^2$, meaning $\sqrt{2} = m/n$.

If there are two integers m, n then $\gcd(m,n) = 1$

Drawing new squares,



So $(2(n-m))^2 = (2m-n)^2$

We reach the conclusion that some smaller version of m, n must exist which means there is no version of m, n that can ever exist.

(3.) (a) $\frac{29}{16} = 1 + \frac{13}{16} \to 1, \frac{16}{13} = 1 + \frac{3}{13}, \to 1, 1, \frac{13}{3} = 4, \; 1, 1, 4 \to \frac{1}{3} = 3$

$\frac{29}{16} = [1, 1, 4, 3] = 1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{3}}}$

(b.) $\frac{32}{19} = 1 + \frac{13}{19} = 1 + \frac{6}{13}, \; 2 + \frac{1}{6} = 6 \to [1, 1, 2, 6] = \boxed{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{6}}}}$

(4.) $\sqrt{2} + 1 = 1 + \left(1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}}\right) \cdots = 2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}} \cdots$

($\sqrt{2} + 1$) is an irrational number because it is an infinite repeating continued fraction.

We can say that $\sqrt{2}$ is also irrational because it is the same fraction, but minus one.

(5.) $\frac{\sqrt{3}+1}{2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}+1}{2} = 1 + \frac{\sqrt{3}-1}{2} = 1 + \cfrac{1}{\frac{2}{\sqrt{3}-1}} = 1 + \frac{1}{\sqrt{3}+1} = 1 + \frac{1}{2 + (\sqrt{3}-1)}$

We can deduce that $\frac{\sqrt{3}+1}{2}$ is irrational because it contains an infinite repeating fraction.