Farrah Rahman
Can publish on site

1. Suppose that $\sqrt{2}$ is rational. Then there exists coprimes p and q where $q \neq 0$ such that $\sqrt{2} = \frac{p}{q}$. So:

$$\begin{aligned} \sqrt{2} &= \frac{p}{q} \\ 2 &= \frac{p^2}{q^2} \\ 2q^2 &= p^2 \end{aligned}$$

So 2 must divide p^2. This is only possible when 2 also divides p. Then there must exist integer m such that $p = 2m$. It follows that:

$$\begin{aligned} 2q^2 &= p^2 \\ 2q^2 &= (2m)^2 \\ 2q^2 &= 4m^2 \\ q^2 &= 2m^2 \end{aligned}$$

So 2 must divide q. But this is absurd, as p and q are coprime. So the initial assumption must have been incorrect, and $\sqrt{2}$ is irrational.

2. Suppose you have two squares with integer side lengths such that one is half the area of the other. Then the smaller one can be placed twice on the diagonal of the larger one. This leaves 2 squares with integer side lengths that are uncovered, and 1 square with an integer side length that is covered twice. So the covered square's area must be twice the uncovered square's area. These squares then fit the original problem and can be used to construct more squares with integer sides that follow the same properties, which in turn can be used to construct smaller squares with integer sides, etc. But positive integers cannot get indefinitely smaller; therefore, the original two squares cannot exist.

3. (a)

$$\begin{aligned} \frac{29}{16} &= 1 + \frac{13}{16} \\ &= 1 + \frac{1}{1 + \frac{3}{13}} \\ &= 1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{3}}} \end{aligned}$$

(b)

$$\begin{aligned} \frac{32}{19} &= 1 + \frac{13}{19} \\ &= 1 + \frac{1}{1 + \frac{6}{13}} \\ &= 1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{6}}} \end{aligned}$$

4.

$$\begin{aligned}
1+\sqrt{2} &= 2+\frac{\sqrt{2}-1}{1} \\
&= 2+\frac{1}{2+(\sqrt{2}-1)} \\
&= 2+\frac{1}{1+\sqrt{2}} \\
&= 2+\frac{1}{1+2-1+\frac{1}{1+\sqrt{2}}} \\
&= 2+\frac{1}{2+\frac{1}{1+\sqrt{2}}} \\
&= 2+\frac{1}{2+\frac{1}{2+\frac{1}{\cdots}}}
\end{aligned}$$

The fraction keeps including an instance of itself, so by substitution it is infinite and so $1+\sqrt{2}$ is irrational. $\sqrt{2}$ must be irrational, as $1+\sqrt{2}$ is irrational and is the sum of an integer plus $\sqrt{2}$.

5.

$$\begin{aligned}
\frac{\sqrt{3}+1}{2} &= 1+\frac{\sqrt{3}-1}{2} \\
&= 1+\frac{1}{\frac{2}{\sqrt{3}-1}} \\
&= 1+\frac{1}{2+\sqrt{3}+1} \\
&= 1+\frac{1}{2+\sqrt{3}+1}
\end{aligned}$$

(not certain how to proceed)