Hw8

Monday, October 11, 2021 4:15 PM

1) Suppose $\sqrt{2}$ is rational. $\exists\, p, q \in \mathbb{Z}$

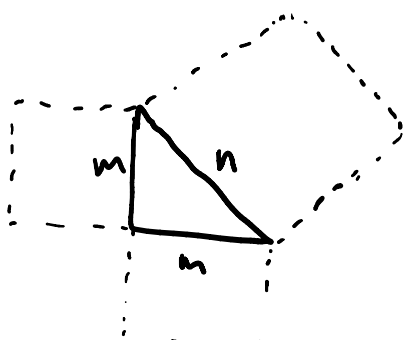
st:

$\sqrt{2} = \frac{p}{q}$, assuming p, q have no common factors.

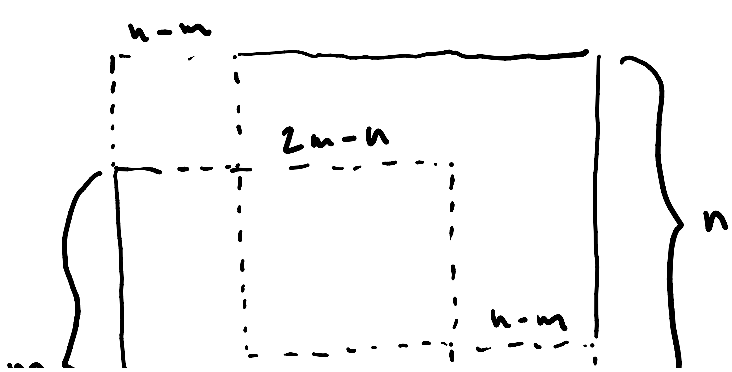
Thus, $2 = \frac{p^2}{q^2} \Rightarrow 2q^2 = p^2$

p^2 is even, p is even. p^2 divisible by 4, but q^2 is also even, q is even. Contradiction, as p and q can't both be even and have no common factors. Thus, $\sqrt{2}$ is irrational

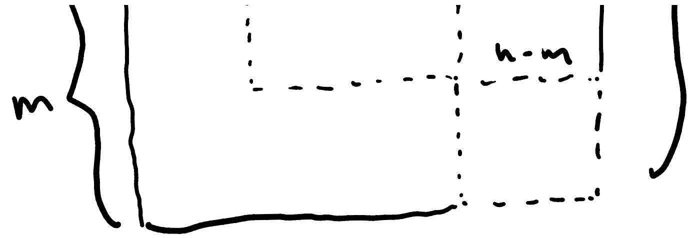
2)



$2m^2 = n^2$, assume $m, n \in \mathbb{Z}$



$2(n-m)^2 = (2m-n)^2$



contradicts that m,n are minimal values, thus no such m,n that exist. $\sqrt{2}$ irrational

3) a) $\frac{29}{16} = 1 + \frac{13}{16}$

$$= 1 + \frac{1}{\frac{16}{13}} = 1 + \frac{1}{1+\frac{3}{13}}$$

$$= 1 + \frac{1}{1+\frac{1}{13/3}} = \boxed{1 + \frac{1}{1+\frac{1}{4+1/3}}}$$

b) $\frac{32}{19} = 1 + \frac{13}{19}$

$$= 1 + \frac{1}{\frac{19}{13}} = 1 + \frac{1}{1+\frac{6}{13}}$$

$$= 1 + \frac{1}{1+\frac{1}{13/6}} = \boxed{1 + \frac{1}{1+\frac{1}{2+1/6}}}$$

4) $\sqrt{2}+1 = 2+(\sqrt{2}-1) = 2 + \frac{1}{\sqrt{2}+1}$

$$= 2 + \frac{1}{2+(\sqrt{2}-1)} = 2 + \frac{1}{2+\frac{1}{1+\sqrt{2}}}$$

$$= 2 + \cfrac{1}{2 + \cfrac{1}{2 + (\sqrt{2} - 1)}} = \boxed{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 \cdots}}}}$$

Thus, it continues forever. $\sqrt{2}+1$ doesn't have a finite continued fraction, and is irrational

5) $$\frac{(\sqrt{3}+1)}{2} = 1 + \left(\frac{\sqrt{3}+1}{2} - 1\right) = 1 + \frac{\sqrt{3}-1}{2}$$

$$= 1 + \cfrac{1}{\cfrac{2}{\sqrt{3}-1}} = 1 + \cfrac{1}{\cfrac{2}{\sqrt{3}-1} \cdot \cfrac{\sqrt{3}+1}{\sqrt{3}+1}}$$

$$= 1 + \cfrac{1}{\cfrac{2\sqrt{3}+2}{2}} = 1 + \cfrac{1}{1+\sqrt{3}}$$

$$= 1 + \cfrac{1}{(\sqrt{3}-1)+1+1} = 1 + \cfrac{1}{(\sqrt{3}-1)+2}$$

$$= 1 + \cfrac{1}{2 + \cfrac{1}{\cfrac{\sqrt{3}-1}{2}}} \cdots$$

Thus, this continued fraction is infinite, and $\frac{\sqrt{3}+1}{2}$ is irrational.