Homework 8

Daniel Rogers
dmr336@scarletmail.rutgers.edu Due 10/10

You may post my answers.

1. Suppose (a,b) integers
   s.t. $\sqrt{2} = a/b$; $2 = a^2/b^2$; $2b^2 = a^2$
   $\exists$ (a,b) w/ a smallest value of a+b
   $(2n)^2 = 2b^2$
   $4n^2 = 2b^2$
   $a^2 = 2b^2$
   b must be an even integer
   $b = 2m$ of some integer $m = b/2$
   $(2m)^2 = 2n^2$
   $m^2 = 2n^2$, so (m,n) is also a possibility
   $m + n = \frac{a+b}{2}$ CONTRADICTION

2. Simplify: $(2b-a)^2 - 2(a-b)^2 \Rightarrow 2b^2 - a^2$
   Algebraic Identity: $(2b-a)^2 - 2(a-b)^2 = -(a^2 - 2b^2)$
   Suppose $\exists$ integers a & b s.t. $a^2 - 2b = 0$
   (a,b) w/ smallest sum (a=b)
   Define $c = 2b - a$, $d = a - b$
   $c^2 - 2d^2 = 0$
   But $c + d = 2b - a + a - b = b < a + b$
   CONTRADICTION

3. a) $29/16$: $29 = 1 \cdot 16 + 13$

$29/16 = ~~1 + 1/(13/16)~~ = 1 + 1/(\frac{16}{13}) = 1 + 1/(1 + 3/13)$

$= 1 + 1/(1 + 1/(4 + 1/3))$

b) $32/19$: $32 = 1 \cdot 19 + 13$

$32/19 = 1 + 1/(19/13) = 1 + 1/(1 + 6/13)$

$= 1 + 1/(1 + 1/(13/6)) = 1 + 1/(1 + 1/(2 + 1/6))$

4. $\sqrt{2} + 1 = 2 + 1/(1 + \sqrt{2}) = 2 + 1/(2 + 1/(1 + \sqrt{2}))$

$= 2 + 1/(2 + 1/(2 + 1/(1 + \sqrt{2})))$

Any rational number may be expressed as a finite continued fraction. $\sqrt{2} + 1$ is thusly irrational, as is $\sqrt{2}$ (since it would begin with $1 + 1/\ldots$ as the only difference.

5. $(\sqrt{3} + 1)/2 = 1 + 1/(\frac{2}{\sqrt{3} - 1}) = 1 + 1/(2 + \frac{3}{\sqrt{3}} - 1)$

$= 1 + 1/(2 + 1/(2 + 2/\sqrt{3} - 1))$ . . .

This is irrational by the same reasoning as the previous question.