Homework 8

1. We will prove by contradiction. Let us assume that sqrt(2) is rational. There must be a way for us to express sqrt(2) as a ratio (p/q) of rational integers such that p and q have no common factors. We can express a resulting equation of sqrt(2) = p/q. Squaring both sides, we have $2 = p^2/q^2$. Rearranging, this gives us $2q^2$ = p^2. p^2 must be even in this instance. The only way the square of a number can be even is if the number itself is also even, so p must be even. This would imply a factor of 4 for p^2 and this means that q^2 and q are also even. Since p and q are both even, they have a common factor and this contradicts our initial assumption. Therefore, sqrt(2) is not rational.
2. If we take a right triangle with base lengths both equal to b and hypotenuse length equal to a, we can construct infinitely small similar triangles and as such can see that sqrt(2) being the ratio of the hypotenuse to the base cannot be rational.

3. Both problems involve the repeated use of Euclid's algorithm for the gcd. The results here are expressed in the form [a_0, a_1, etc.]
    a. 29/16 = [1,1,4,3]
    b. 32/19 = [1,1,2,6]
4. The infinite continued fraction is 2+1/{2+[1/(2+1/…)]} Since there isn't a finite convergence such that we can represent the number sqrt(2)+1 by a ratio of rational numbers, it therefore follows that this number is not rational.
5. The infinite continued fraction is [1,2,1,2,1,2,1…]. Since there isn't a finite convergence such that we can represent the number by a ratio of integers, it therefore follows that this number is not rational