Homework 8

1. If $\sqrt{2}$ is rational

$\sqrt{2} = a/b$

$2 = a^2/b^2$

$a^2 = 2 \cdot b^2$

$a^2$ is even $\Rightarrow$ $a$ is even

let $a = 2k$

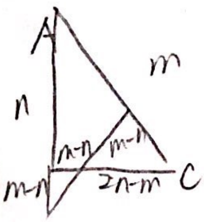
$2 = (2k)^2/b^2$

$= 4k^2/b^2$

$b^2 = 2k^2$; $b^2$ is even, $b$ is even.

If both a and b are even, then it is contradict to the original statement

2. Assume $m/n = \sqrt{2}$




$(m,n) = 1$

$\sqrt{2} = \frac{m}{n} = \frac{m(\sqrt{2}-1)}{n(\sqrt{2}-1)} = \frac{2n-m}{m-n}$

Since $1 > \sqrt{2}-1 > 0$

$n > m-n$.

But n is the least value,

Proved

3(a) $\frac{29}{16} = 1+\frac{13}{16} = 1+\frac{1}{1+\frac{3}{16}} = 1+\frac{1}{1+\frac{1}{\frac{16}{3}}} = 1+\frac{1}{1+\frac{1}{5+\frac{1}{3}}}$

(b) $\frac{32}{19} = 1+\frac{13}{19} = 1+\frac{1}{1+\frac{6}{13}} = 1+\frac{1}{1+\frac{1}{\frac{13}{6}}} = 1+\frac{1}{1+\frac{1}{2+\frac{1}{6}}}$

4. Suppose $\sqrt{2}+1$ is rational

$\sqrt{2}+1 = a/b$

$\sqrt{2} = a/b - 1$

$\sqrt{2} = \frac{a-b}{b}$ $\Rightarrow$ a and b are both integer, $\sqrt{2}$ is rational

Which is a contradictione

Therefore. $\sqrt{2}+1$ is irrational

5. Suppose $(\sqrt{3}+1)/2$ is a rational number

$$\frac{\sqrt{3}+1}{2} = \frac{a}{b}$$

$$\sqrt{3}+1 = 2a/b$$

$$\sqrt{3} = 2a/b - 1$$

$$\sqrt{3} = \frac{2a-b}{b}$$

Both a and b are integers, $\sqrt{3}$ is rational, which is a contradiction

Therefore, $(\sqrt{3}+1)/2$ is irrational