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It is OK to post the homework in your website.

1. $19 = 1 \times 14 + 5$, so $\gcd(19, 14) = \gcd(14, 5)$ and $5 = 19 - 14$

$$14 = 5 \times 3 - 1, \quad 1 = 5 \times 3 - 14 = (19 - 14) \times 3 - 14 = 19 \times 3 - 14 \times 4$$

So give the cashier three 19-dollar, and the cashier return four 14-dollar.

2. $109 = 95 \times 1 + 14$, so $\gcd(109, 95) = \gcd(95, 14)$, $14 = 109 - 95$

$$95 = 14 \times 7 - 3, \text{ so } 3 = 14 \times 7 - 95, \quad 14 = 3 \times 4 + 2,$$

$$\text{and } 3 = 1 + 2 \times 1, \text{ so } 1 = 3 - 2 = (14 \times 7 - 95) - (14 - 3 \times 4)$$

$$= 14 \times 6 - 95 + (14 \times 7 - 95) \times 4$$

$$= 14 \times 6 - 95 + 14 \times 28 - 95 \times 4$$

$$= 14 \times 34 - 95 \times 3$$

$$= (109 - 95) \times 34 - 95 \times 3$$

$$= 109 \times 34 - 95 \times 39$$

So give the cashier thirty-four 109-dollar, and the cashier return thirty-nine 95-dollar.

3. $37 = 16 \times 2 + 5$, so $\gcd(37, 16) = \gcd(16, 5)$, and $5 = 37 - 16 \times 2$

$$16 = 5 \times 3 + 1, \quad 1 = 16 - 5 \times 3 = 16 - (37 - 16 \times 2) \times 3$$

$$= 16 - 37 \times 3 + 16 \times 6 = 16 \times 7 - 37 \times 3$$

So put seven 16-kg on left side, and put three 37-kg on right side, and we can measure 1 kg of coffee on the right side.