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1. coin worth 19 } pay for 1 dollar coffee.
coin worth 14 }

$19 = 1\cdot 14 + 5 \rightarrow 5 = 19 - 14$ and $\gcd(19,14) = \gcd(14,5)$

$14 = 2\cdot 5 + 4 \rightarrow 4 = 14 - 2\cdot 5$ and $\gcd(14,5) = \gcd(5,4)$

$= 14 - 2\cdot(19-14)$

$= 3\cdot 14 - 2\cdot 19$

$5 = 4\cdot 1 + 1 \rightarrow 1 = 5 - 4$ and $\gcd(5,4) = \gcd(4,1)$

$1 = (19-14) - (3\cdot 14 - 2\cdot 19)$

$1 = 3\cdot 19 - 4\cdot 14$

Therefore, hand the cashier 3 19-dollar bills and take 4 14-dollar bills for change.

2. 2 types of coins → \$109 and \$15. Pay for \$1 coffee.

$109 = 7\cdot 15 + 4 \rightarrow 4 = 109 - 7\cdot 15$ and $\gcd(109,15) = \gcd(15,4)$

$15 = 3\cdot 4 + 3 \rightarrow 3 = 15 - 3\cdot 4$ and $\gcd(15,4) = \gcd(4,3)$.

$3 = 15 - 3(109 - 7\cdot 15)$

$3 = 22\cdot 15 - 3\cdot 109$

$4 = 1\cdot 3 + 1 \rightarrow 1 = 4 - 3$ and $\gcd(4,3) = \gcd(3,1)$.

$1 = (109 - 7\cdot 15) - (22\cdot 15 - 3\cdot 109)$

$1 = 4\cdot 109 - 29\cdot 15$

Therefore, hand the cashier 4 109-dollar bills and expect 29 15-dollar bills as change.

3. 37 kg and 16 kg weights. How to measure one kg?

$37 = 2 \cdot 16 + 5 \rightarrow 5 = 37 - 2 \cdot 16$ $\quad gcd(37,16) = gcd(16,5)$

$16 = 3 \cdot 5 + 1 \rightsquigarrow 1 = 16 - 3 \cdot 5$ $\quad gcd(16,5) = gcd(5,1)$

$= 16 - 3 \cdot (37 - 2 \cdot 16)$

$1. = 7 \cdot 16 - 3 \cdot 37$

Therefore, put 7 16 kg weights on one side and 3 37 kg weight on the second side. The second side will be lighter than the first by 1kg. Add coffee until the sides balance, and then you would have weighted 1kg of coffee.

4. $x \bmod 21 = 5$ and $x \bmod 25 = 8$

Apply Euclidean Extended Algorithm to $n_1 = 21$, $n_2 = 25$.

Then:

$25 = 1 \cdot 21 + 4 \rightsquigarrow 4 = 25 - 21$ $\quad gcd(25,21) = gcd(21,4)$

$21 = 5 \cdot 4 + 1 \rightsquigarrow 1 = 21 - 5 \cdot 4$ $\quad gcd(21,4) = gcd(4,1)$

$1 = 21 - 5(25 - 21)$

$\underline{1 = 6 \cdot 21 - 5 \cdot 25}$

Take modulo 21 of $6 \cdot 21 - 5 \cdot 25 = 1$:

$(-5) \cdot 25 \equiv 1 \pmod{21}$

$-5 = 25^{-1} \pmod{21}$

$16 = 25^{-1} \pmod{21}$

$*$ here add 21 to both sides to get rid of -5.

so $25^{-1} \pmod{21} = 16$

Take modulo 25 of $6 \cdot 21 - 5 \cdot 25 = 1$:

$$6 \cdot 21 \equiv 1 \pmod{25}$$

$$6 = 21^{-1} \pmod{25}$$

So $21^{-1} \pmod{25} = 6$.

Now plug in the values $a_1 = 5$, $a_2 = 8$, $n_1 = 21$, $n_2 = 25$

$x = a_1 n_2 \cdot (n_2^{-1} \pmod{n_1}) + a_2 n_1 \cdot (n_1^{-1} \pmod{n_2}) \pmod{n_1 n_2}$

$x = 5 \cdot 25 \cdot \underbrace{(25^{-1} \pmod{21})} + 8 \cdot 21 \underbrace{(21^{-1} \pmod{25})} \pmod{21 \cdot 25}$

$x = 5 \cdot 25 \cdot 16 + 8 \cdot 21 \cdot 6 \pmod{525}$

$x = 3008 \pmod{525}$

$= 3008 - 5 \cdot 525$ ~~(mod 525)~~

$= 383$

Therefore $\underline{x = 383}$.

Check:

$383 \pmod{21} = (383 - 18 \cdot 21) = 5$ ✓

$383 \pmod{25} = 383 - 15 \cdot 25 = 8$ ✓

Chapter V

1. Roman Empire
   → East stimulated the static civilization of the West
   → rupture

Germanic kingdoms
- social institutions + intellectual life remained the same
- central authority after 476 (fall of Western Empire)
  → shared by Emperor in Constantinople and the popes of Rome
- Anicius Manilius Severinus Boetius
  → wrote mathematical texts
  ↳ survived because author died as a martyr in 524
  - 'Institutio arithmetica'
    - Pythagorean number theory
    - part of ancient trivium and quadrivium.
      arithmetic, geometry, astronomy and music

Arabs made relations between the Near Orient and the Christian Occident difficult
→ Frankish Gaul and other parts of the Roman Empire
  - large scale economy vanished
    → reduced to a state of semi-barbarism
- North Frankish landlords under the Carolingians
  - crowning of Charlemagne in 800 as Emperor of the Holy Roman Empire
    → papacy allied with Carolingians

2. Western feudalism - little appreciation of math
→ mainly use math for computation of Easter time
Ecclesiastical mathematicians:
→ Alcuin - Problems for 'the Quickening of the Mind'
~ associated with Charlemagne court
→ Gerbert (pope under the name Sylvester II)
→ studied Arabic mathematics

3. Development of Western cities
→ no means of obtaining a vast supply of slaves
- self-governing units
- fight against feudal lords ~ victorious in 12^{th}-14^{th} century
- rapid expansion of trade and money economy
- gradual improvement of technology
- support from small princes
⇒ First national states in Western Europe
- established commercial relations
- 1085 - Toledo taken from the Moors
→ students flocked to learn Arabic sciences

4. Powerful commercial cities in Italy ~12^{th}-13^{th} century
→ Genoa, Pisa, Venice, Milan, Florence
Leonardo (Fibonacci or 'son of Bonaccio')
→ 'Liber Abaci' - arithmetical + algebraic information
→ 'Practica Geometriae'
- quotes 'Al-Khwārizmī → $x^2+10x=39$.
→ leads to the 'series of Fibbonacci'

Leonardo (cont)

→ roots of $x^3+2x^2+10x=20$ cannot be expressed as $\sqrt{a+\sqrt{b}}$

Introduction of numerals was slow

- 'Codex Vigilanus' written in Spain in 976
- earliest French manuscript from 1275

Computation performed on ancient abacus

→ a board with counters or pebbles.

Roman numerals to registrate the results of computations

Statutes of the 'Arte del Cambio' (1299) - bankers of Florence were forbidden to use Arabic numerals and were obliged to use cursive Roman ones

5. Expansion of trade → interest in mathematics
   - → practical interest, taught by self-made masters
   - → speculative mathematics cultivated by philosophers
     - → speculations on the nature of motion, of the continuum and of infinity
     - → Origen denied infinity (following Aristotle)
     - → St. Augustine in the 'Civitas Dei' accepted the sequence of integers as infinity.
     - → St. Thomas Aquinas
       - → accepted Aristotle's 'infinitum actu non datur'
         - → every continuum as potentially divisible ad infinitum
           - → point not a part of a line.
   - → Thomas Bradwardine → star polygons
   - → Nicole Oresme → fractional powers: $8=4^{1\frac{1}{2}}$ → $\boxed{\frac{p.1}{1.2}}$ 4

→ Nicole Oresme, Bishop of Lisieux in Normandy
→ 'De latitudinibus formarum'
→ transition to modern coordinate geometry

6. Mercantile cities
→ interested in counting, in arithmetic, in computation
→ 'Rechenhaftigkeit' → by Sombart
→ interest of 15^{th}, 16^{th} centuries

Johannes Müller of Königsberg
→ computer, instrument maker, printer and scientist
→ translate + publish classical math manuscripts
→ continued translation of Ptolemy
→ translate Apollonois, Heron and Archimedes
→ 'De triangulis omnimodus' ~ intro to trig
→ influenced further development