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Dr. Z, History of Math
10/3/21

Homework for Lecture 7 - do not post

① We start by finding the gcd(19, 14)

$$19 = 14(1) + 5$$

$$14 = 5(2) + 4$$

$$5 = 4(1) + 1$$

We rewrite 1 as

$$1 = 5 - 4(1)$$

$$1 = 5 + 4(-1)$$

$$1 = 5 + (14 - 5(2))(-1) \quad \text{since } 14 = 5(2) + 4$$

$$1 = 5 - 14 + 5(2)$$

$$1 = 5(3) - 14$$

$$1 = (19 - 14(1))(3) - 14 \quad \text{since } 19 = 14(1) + 5$$

$$1 = 19(3) - 14(3) - 14$$

$$1 = 19(3) - 14(4)$$

I give the cashier three 19 dollar coins, and the cashier returns four 14 dollar coins to me.

② We start by finding the gcd(109, 95)

$$109 = 95(1) + 14$$

$$95 = 14(6) + 11$$

$$14 = 11(1) + 3$$

$$11 = 3(3) + 2$$

$$3 = 2(1) + 1$$

We rewrite 1 as

$$1 = 3 - 2(1)$$

$$1 = 3 + 2(-1)$$

$$1 = 3 + (11 - 3(3))(-1) \quad \text{since } 11 = 3(3) + 2$$

$$1 = 3 - 11 + 3(3)$$

$$1 = 3(4) - 11$$

$$1 = (14 - 11(1))(4) - 11 \quad \text{since } 14 = 11(1) + 3$$

$$1 = 14(4) - 11(4) - 11$$

$$1 = 14(4) + 11(-5)$$

$$1 = 14(4) + (95 - 14(6))(-5) \quad \text{since } 95 = 14(6) + 11$$

$$1 = 14(4) + 95(-5) + 14(30)$$

$$1 = 14(34) + 95(-5)$$

$$1 = (109 - 95)(34) + 95(-5) \quad \text{since } 109 = 95(1) + 14$$

$$1 = 109(34) - 95(39)$$

I give the cashier thirty four 109 dollar coins, and the cashier returns thirty nine 95 dollar coins to me.

③ We start by finding the gcd(37, 16)

$$37 = 16(2) + 5$$

$$16 = 5(3) + 1$$

We rewrite 1 as

$$1 = 16 - 5(3)$$

$$1 = 16 + 5(-3)$$

$$1 = 16 + (37 - 16(2))(-3) \quad \text{since } 37 = 16(2) + 5$$

$$1 = 16 + 37(-3) + 16(6)$$

$$1 = 16(7) - 37(3)$$

I would put seven 16 kg weights on one side and put three 37 kg weights on the other side in order to measure the 1 kg of coffee.