Quin Boob
HW 7

1) $\gcd(19, 14)$

$19 = 1 \times 14 + 5$

$\gcd(14, 5) \qquad 5 = 19 - 1 \times 14$

$14 = 2 \times 5 + 4$

$\gcd(5, 4) \qquad 4 = 14 - 2 \times 5 = 14 - 2(19 - 14) = 3 \times 14 - 2 \times 19$

$5 = 1 \times 4 + 1$

$\gcd(4, 1)$

$1 = 5 - 1 \times 4 = (19 - 1 \times 14) - 1(3 \times 14 - 2 \times 19)$

$1 = 19 - 1 \times 14 - 3 \times 14 + 2 \times 19$

$1 = 3 \times 19 - 4 \times 14$

you pay with 3 $19 coins and recieve 4 $14 coins in return

2) $\gcd(109, 95)$

$109 = 1(95) + 14$

$\gcd(95, 14) \qquad 14 = 109 - 1(95)$

$95 = 6(14) + 11$

$\gcd(14, 11) \quad 11 = 95 - 6(14) = 95 - 6(109 - 1(95)) = 7(95) - 6(109)$

$14 = 1(11) + 3$

$\gcd(11, 3) \qquad 3 = 14 - 1(11) = (109 - 1(95)) - 1(7(95) - 6(109))$

$11 = 3(3) + 2 \qquad\qquad\qquad\qquad = 7(109) - 8(95)$

$\gcd(3, 2) \qquad 2 = 11 - 3(3) = 7(95) - 6(109) - 3(7(109) - 8(95))$

$3 = 1(2) + 1 \qquad\qquad\qquad\qquad = 31(95) - 27(109)$

$\gcd(2, 1) \qquad 1 = 3 - 1 \times 2 = 7(109) - 8(95)) - (31(95) - 27(109)) = 34(109) - 39(95)$

You pay with 34 $109 coins and recieve 39 $95 coins in return

3) $\gcd(37,16)$

$\quad 37 = 2(16) + 5$

$\gcd(16,5) \qquad 5 = 37 - 2(16)$

$\quad 16 = 3(5) + 1$

$\gcd(5,1) \qquad 1 = 16 - 3(5) = 16 - 3(37 - 2(16)) = 7(16) - 3(37)$

On one side of the balance you will put 7 16 kg weights and on the other you will put 3 37 kg weights. Both sides are offset by 1 kg. To weigh the coffee you will place it on the side with the 37 kg weights.

4) $X = \left(5(25)\left(\frac{1}{25} \bmod 25\right) + 8(21)\left(\frac{1}{21} \bmod 25\right)\right) \bmod (21)(25)$

$X = \left(5(25)(1/25) + 8(21)^{1}/_{21}\right) \bmod 525$

$X = 13 \bmod 525$

$X = 13 \quad$ I think