

①  $d = \gcd(19, 14) \Rightarrow d = 19m + 14n$

$19 = 1 \cdot 14 + 5 \Rightarrow 5 = 1 \cdot 19 - 1 \cdot 14$

$14 = 2 \cdot 5 + 4 \Rightarrow 4 = 1 \cdot 14 - 2 \cdot 5$

$\Rightarrow 4 = 3 \cdot 14 - 2 \cdot 19$

$\rightarrow \gcd(5, 4) = \gcd(4, 1) \Rightarrow 5 = 1 \cdot 4 + 1$   
 $1 = 1 \cdot 5 - 1 \cdot 4$

$1 = 19 - 14 - (3 \cdot 14 - 2 \cdot 19)$

$1 = 3(19) - 4(14)$

②  $d = \gcd(109, 95) \Rightarrow d = 109m + 14n$

$109 = 1 \cdot 95 + 14 \Rightarrow \gcd(109, 95) = \gcd(95, 14) = \gcd(14, 11)$

$95 = 6 \cdot 14 + 11$

$14 = 1 \cdot 109 - 1 \cdot 95$

$11 = 1 \cdot 95 - 6 \cdot 14$

$11 = 95 - 6(109 - 95)$

$11 = 7(95) - 6(109)$

$14 = 1 \cdot 11 + 3$

$\Rightarrow 3 = 1 \cdot 14 - 1 \cdot 11$

$= 109 - 95 - (7(95) - 6(109))$

$3 = 7(109) - 8(95)$

$11 = 3 \cdot 3 + 2$

$\Rightarrow 2 = 1 \cdot 11 - 3 \cdot 3$

$2 = (7(95) - 6(109)) - 3(7(109) - 8(95))$

$= 7(95) - 6(109) - 21(109) + 24(95)$

$= 31(95) - 27(109)$

$3 = 2 \cdot 1 + 1 \Rightarrow \gcd(3, 2) = \gcd(2, 1)$

$1 = 1 \cdot 3 - 1 \cdot 2$

$1 = 7(109) - 8(95) - (31(95) - 27(109))$

$1 = 7(109) - 8(95) - 31(95) + 27(109)$

$1 = 34(109) - 39(95)$

③  $d = \gcd(37, 16) \Rightarrow d = 37m + 16n$

$37 = 2 \cdot 16 + 5 \Rightarrow \gcd(37, 16) = \gcd(16, 5) = \gcd(5, 1)$

$16 = 3 \cdot 5 + 1$

$5 = 1 \cdot 37 - 2 \cdot 16$

$1 = 1 \cdot 16 - 3 \cdot 5$

$\Rightarrow 1 = 1 \cdot 16 - 3(37 - 2 \cdot 16)$

$= 16 - 3(37) + 6(16)$

$1 = 7(16) - 3(37)$