

## Homework 7

①  $d = \gcd(19, 14)$  and  $d = 19m + 14n$

$$19 = 1 \cdot 14 + 5, \quad \gcd(19, 14) = \gcd(14, 5), \quad 5 = 1 \cdot 19 - 1 \cdot 14$$

$$14 = 2 \cdot 5 + 4, \quad \gcd(14, 5) = \gcd(5, 4), \quad 4 = 1 \cdot 14 - 2 \cdot 5$$

$$4 = 1 \cdot 14 - (2 \cdot (19 - 14))$$

$$4 = 3 \cdot 14 - 2 \cdot 19$$

$$5 = 1 \cdot 4 + 1, \quad \gcd(5, 4) = \gcd(4, 1) \quad 1 = 1 \cdot 5 - 1 \cdot 4$$

$$1 = 19 - 14 - (3 \cdot 14 - 2 \cdot 19)$$

$$1 = 19 - 14 - 3(14) + 2(19)$$

$$1 = 3(19) - 4(14)$$

②  $d = \gcd(109, 95)$  and  $d = 109m + 14n$

$$109 = 1 \cdot 95 + 14, \quad \gcd(109, 95) = \gcd(95, 14), \quad 14 = 1 \cdot 109 - 1 \cdot 95$$

$$95 = 6 \cdot 14 + 11, \quad \gcd(95, 14) = \gcd(14, 11), \quad 11 = 1 \cdot 95 - 6 \cdot 14$$

$$11 = 95 - 6(109 - 95)$$

$$11 = 95 - 6(109) + 6(95)$$

$$11 = 7(95) - 6(109)$$

$$14 = 1 \cdot 11 + 3, \quad \gcd(14, 11) = \gcd(11, 3), \quad 3 = 1 \cdot 14 - 1 \cdot 11$$

$$3 = 109 - 95 - (7(95) - 6(109))$$

$$3 = 7(109) - 8(95)$$

$$11 = 3 \cdot 3 + 2, \quad \gcd(11, 3) = \gcd(3, 2), \quad 2 = 1 \cdot 11 - 3 \cdot 3$$

$$2 = (7(95) - 6(109)) - 3(7(109) - 8(95))$$

$$2 = 7(95) - 6(109) - 21(109) + 24(95)$$

$$2 = 31(95) - 27(109)$$

$$3 = 2 \cdot 1 + 1, \quad \gcd(3, 2) = \gcd(2, 1), \quad 1 = 1 \cdot 3 - 1 \cdot 2$$

$$1 = 7(109) - 8(95) - (31(95) - 27(109))$$

$$1 = 7(109) - 8(95) - 31(95) + 27(109)$$

$$(3) \quad d = \gcd(37, 16), \quad d = 37m + 16n$$

$$37 = 2 \cdot 16 + 5, \quad \gcd(37, 16) = \gcd(16, 5), \quad 5 = 1 \cdot 37 - 2 \cdot 16$$

$$16 = 3 \cdot 5 + 1, \quad \gcd(16, 5) = \gcd(5, 1), \quad 1 = 1 \cdot 16 - 3 \cdot 5$$

$$1 = 1 \cdot 16 - 3(37 - 2 \cdot 16)$$

$$1 = 16 - 3(37) + 6(16)$$

$$1 = 7(16) - 3(37)$$