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1. $1 = 19x + 14y$

$19 = 14 \cdot 1 + 5$

$14 = 5 \cdot 2 + 4$

$5 = 4 \cdot 1 + 1$

$1 = 5 + 4(-1)$

$4 = 14 + 5(-2)$

$1 = 5 + (-1)(14 + 5(-2))$

$1 = 5 + (-1)(14) + (2)(5)$

$1 = 3(5) + (-1)(14)$

$19 + 14(-1) = 5$

$1 = 3(19 + (14)(-1)) + (-1)(14)$

$1 = 3(19) + (-3)(14) + (-1)(14)$

$1 = 3(19) + (-4)(14)$

You give 3-19 dollar bills and they give you back 4-14 dollar bills.

2. $1 = 109x + 95y$

$109 = 95 \cdot 1 + 14$

$95 = 14 \cdot 6 + 11$

$14 = 11 \cdot 1 + 3$

$11 = 3 \cdot 3 + 2$

$3 = 2 \cdot 1 + 1$

$1 = 3 + 2(-1)$

$11 + 3(-3) = 2$

$1 = 3 + (-1)(11 + 3(-3))$

$1 = 4(3) + (-1)(11)$

$14 + 11(-1) = 3$

$1 = 4(14 + 11(-1)) + (-1)(11)$

$1 = (5)(11) + 4(14)$

$95 + 14(-6) = 11$

$1 = (-5)(95 + (14)(-6)) + 4(14)$

$1 = 34(14) + (-5)(95)$

$109 + 95(-1) = 14$

$1 = 34(109 + 95(-1)) + (-5)(95)$

$1 = 34(109) + (-39)(95)$

You give 34 - 109 dollar bills and they give back 39 - 95 dollar bills.

3. $1 = 37x + 16y$

$37 = 16 \cdot 2 + 5$

$16 = 5 \cdot 3 + 1$

$1 = 16 + 5(-3)$

$37 + 16(-2) = 5$

$1 = 16 + (37 + 16(-2))(-3)$

$1 = 7(16) + (-3)(37)$

Put 7-16 weights on one side then 3-37 weights plus the one bag of coffee on the other side to balance them out.

4. $x = 5 \bmod 21$

$x = 8 \bmod 25$

b_i	N_i	x_i	$b_i N_i x_i$	Σ
5	25	16	2000	3008
8	21	6	1008	

$25x_1 \equiv 1 \bmod 21 \rightarrow x_1 = 16$

$21x_2 \equiv 1 \bmod 25 \rightarrow x_2 = 6$

$x = 3008 + 525k, \quad k \in \mathbb{Z}$

Smallest positive integer: $x = 383$