Homework 6 - History of Math

Karan Amin

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Problem 1

The fifth postulate seems like it should be a theorem instead of an axiom, and many mathematicians tried to prove this was the case but were unsuccessful. It cannot be proven from the other axioms because it is independent from the other axioms. If we were to ignore the fifth postulate and assume it's false, then entirely new geometries emerged which new meanings to things like points and lines.

Problem 2

Godel showed that there exists true statements that cannot be proved. His proof does this by making paradoxical statements. His incompleteness theorems made it so that we cannot ever have a "theory of everything" using one system. Whatever we can prove depends on the starting assumptions made. He first mapped any possible mathematical statement to a unique "Godel number". There were finite symbols 1 to 12. Then any combination of these symbols and variables got its own Godel number using those original Godel numbers. Then he uses this foundation to show that if a set of axioms is consistent, then its incomplete, and then shows that no set of axioms can prove its own consistency.

Problem 3

Professor Z says that Godel and Turing didn't prove that "there exist true yet unprovable statements", and instead showed via a metaprove that many statements that were thought to be interesting were in fact not. The simple assumption of an "infinite tape" is preposterous since there is no such thing as an infinite piece of tape. Similarly the halting problem is meaningless since it is the same question as "does there exist an integer N such that the program halts in less than N steps?" And since it assumes N can be any number and there are infinite positive numbers, it must be nonsense. Dr. Z gives the proper

definition of a Turing machine as a computer with at most M memory, therefore it's finite, and then ask the question, "does it halt in less than N steps"? He believes we should view the world in terms of finite things.

Problem 4

i) The only scenario is that either A is telling the truth and B is lying or B is lying and A is telling the truth.

ii) The only scenario for this is that A is lying and B is telling the truth.

iii) There is no situation where this system is consistent.

iv) There is only two scenarios. Either A and C are telling the truth and B and D are lying, or A and C are lying and B and D are telling the truth.