Anna Zhebrun
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Homework 6 10/02/2021

1. The parallel axiom is independent of the other four axioms and holds only for certain geometries. Meanwhile, non-Eucledian geometries exist where the 1^{st} 4 axioms hold, but the Parallel axiom doesn't. If the axiom could be proved from the first 4 it would hold everywhere. But since it holds for only Eucledian geometry it is its own axiom.

2. To prove that there exist true but unprovable mathematical statements, Godel considered some statement S = 'This statement is unprovable.' There are two possibilities:
   1. It is provable, then S is false. But then one has to prove a falsehood, which should be impossible.
   2. The statement is unprovable, then S is true.

Therefore, Godel showed that some statements will be both true and unprovable.

3. Dr Z.'s version of Godel's famous theorem states (as far as I understood) that 'many statements that were believed to be meaningful, are really utterly devoid of meaning.' Therefore, doing away with many senseless concepts, such as infinity, would improve our understanding and advancement of mathematics.

4. i) A: B is a Lie Teller

B: A is a Lie Teller

Possible (A, B) -> (T, F) and (F, T)

ii) A: B is a Truth teller

B: A is a Lie teller

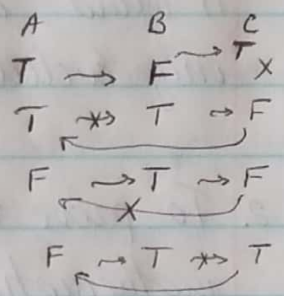
No possible scenarios.

iii) A: B is a Lie Teller

B: C is a lie teller

C: A is a lie teller

No possible scenarios



(iv) A: B is a Lie teller

B: C is a Lie teller

C: D is a lie teller

D: A is a lie teller

Possible (A, B, C, D) ⇒ (T, F, T, F) and (F, T, F, T).

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Chapter IV The Orient after the Decline of Greek Society

1. Oriental + Greek influences → sciences of Alexandria, Constantinople and India.

395 - Theodosis I founded the Byzantine Empire
→ Greek capital Constantinople
→ but Greeks only a section of urban populations
- guardian of Greek culture, bridge between the Orient and the Occident

622 - year of Hegira
- Arabs conquered large sections of Western Asia
- disappearance of political hegemony of the Greeks.
- replace Greek-Roman civilization by Islam
- despite use of Arab language, ancient cultures and traditions remained.

2. Decline of Roman empire
→ shift of mathematical research to India, then to Mesopotamia.

Indian contributions - the 'Siddhāntas'
→ Sūrya Siddhānta - table of sines
→ books about astronomy, epicycles, sexagesimal fractions
- schools centered in Ujjain (Central India) and Mysore (South)

- Āryabhata ('the first', c.500) and Brahmagupta (c. 625)
  - arithmetical-algebraic parts → $\pi = 3.1416$
  - $ax + by = c$ (where a, b, c are integers)

- Mahāvīra - rational triangles and quadrilaterals

→ 'Lilāvati' – arithmetic + mensuration

$x = 50$

↑ $x = -5$

- Bhāskara from Ujjain – solved $x^2 - 45x = 250$

Hindus accepted only integer solutions

→ admitted negative roof of equations (advance beyond Diophantos).

→ present decimal position system

→ on a plate of the year 595 A.D.

→ 'sunya' = zero repr.

- 'Bakshali' manuscript

- seventy leaves of birch (3-12th century A.D.)

- dot to express zero

- 9th century – epigraphic record with a sign for zero

- 662 – Severus Sēbōkht

- oldest definite reference to Hindu place value system

Two types of symbols used to express place value numerals

- Hindu symbols by Eastern Arabs

- 'gobâr' in Spain and by Western Arabs

4. Sassanian Persia

- Babylon replaced by Seleucia-Ktesiphon

- Arabic conquest (641) – mostly unaffected

- Arabic replaced Pehlevi

- shiism → only accepted modified Islam

Abbāsid ~~caliphs~~ caliphs: Al-Mansur (754-775), Hārūn al-Rashid (786-809), Al-Ma'mūn (813-833)

→ promote astronomy + mathematics

→ 'House of Wisdom' – library at Bagdad

Muhammad ibn Mūsā al-Khwārizmī (825)

- books on math + astronomy
- explain Hindu system of numeration
  - Latin translation: "Algoritmi de numero Indorum"
    - → 'algorithms' as a term
- 'Hisab al-jabr wal-muqābala'
  - → 'science of reduction and cancellation'
  - → 'algebra' from al-jabr
    - → middle of 19th century just science of equations
  - ~ several equations:

    $x^2 + 10x = 39$ $x^2 + 21 = 10x$ $3x + 4 = x^2$

    ↳ runs like a 'thread of Gold'
- more Oriental than Greek influences
  - lack of an axiomatic foundation

5. Arabic scholars translated Greek classics into Arabic
   - → stress computational and practical side of mathematics
   - → downplay theoretical
   - → interested in trig
     - 'sinus' from Sanscrit jyā
       - ↳ half the cord of the double arc

Al-Battānī - table of cot for every degree
- cosine rule for spherical triangle

Abū'l-Wafā' - sine theorem of spherical trig
- introduce secant and cosecant

Al-Karkhi - $\sqrt{8} + \sqrt{18} = \sqrt{50}$ $\sqrt[3]{54} - \sqrt[3]{2} = \sqrt[3]{16}$

6. 1000 B.C. - Saljūq (seldiuk) Turks
- Omar Khayyam → 'Rubaiyat'
- reform of old Persian calendar
- 'Algebra' - investigation of cubic equation

1256 - Bagdad by Mongols
- observatory of Maragha built by Hulagu for Nasir al-din
↳ separate trig
↳ attempts to prove Euclid's parallel axioms

Egypt - Ibn Al-Haitham - greatest Muslim physicist
→ 'Optics'
→ solved the 'problem of Alhazen'
→ biquadratic equ

Spain at Cordoba - Al-Zarqāli
→ editor of Toledan planetary tables

China:
- Tsu Ch'ung Chi - $\pi = \frac{355}{113}$
- mathematicians of T'ang dynasty (610-907)
- Ten Classics
- Chin Chiu-Shao (13^{th} century) - solns of higher degree eqns.

$$x^4 - 763200x^2 + 40642560000 = 0$$

→ similar to Horner's method