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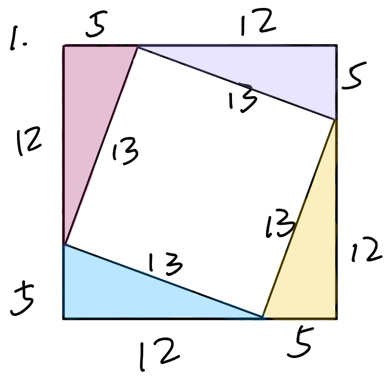
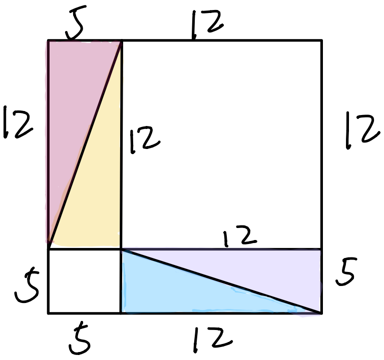
It is OK to post the homework in your website.

1.





$c^2+4\cdot(ab/2)$

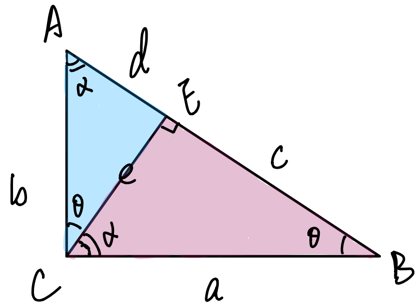
$a=12,\ b=5,\ c=13$

$a^2+b^2+4\cdot(ab/2)$

$a=12,\ b=5,\ c=13$

From the digrams, we can see that $5^2+12^2=13^2$ that is $a^2+b^2=c^2$.

2.



Proof. Since $\angle CDB = \angle ACB = 90°$,
$\angle CAB$ and $\angle CAE$ are the same angle,
$\angle ACE = \angle EBC$

Then $\triangle ABC \sim \triangle ACE$.

Since $\angle CBA$ and $\angle CBE$ are the same angle,
$\angle EBC = \angle CAB$

Then $\triangle ABC \sim \triangle CBE$.

Then $\triangle ACE \sim \triangle CBE$.

So $\dfrac{BC}{AB} = \dfrac{BE}{CB}$, $\dfrac{AC}{AB} = \dfrac{AE}{AC}$,

then we have $\dfrac{a}{d+c} = \dfrac{c}{a}$, $\dfrac{b}{d+c} = \dfrac{d}{b}$.

So $a^2 = c^2+cd$, $b^2 = d^2+dc$,

then $a^2+b^2 = c^2+2cd+d^2 = (c+d)^2$

3. $a = 2mn$, $b = m^2 - n^2$, $c = m^2 + n^2$, $a = 120$

$mn = 60$

$m = 60, n = 1$, $a = 120$, $b = 3599$, $c = 3601$ ✓

$m = 30, n = 2$, $a = 120$, $b = 896$, $c = 904$ ✗

$m = 20, n = 3$, $a = 120$, $b = 391$, $c = 409$ ✓

$m = 15, n = 4$, $a = 120$, $b = 209$, $c = 241$ ✓

$m = 12, n = 5$, $a = 120$, $b = 119$, $c = 169$ ✓

$m = 10, n = 6$, $a = 120$, $b = 64$, $c = 136$ ✗

Answer: $(120, 3599, 3601)$, $(120, 391, 409)$, $(120, 209, 241)$
$(120, 119, 169)$

4. Proof. For example, $(3d)^2 + (4d)^2 = (5d)^2$, where d is a positive integer.
So $d = 1, 2, 3, \cdots$, there are infinitely d.
So there are infinitely triples like $(3d, 4d, 5d)$

5. Pierre de Fermat

6. Carl Friedrich Gauss

7. Pierre de Fermat

8. Andrew Wiles