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Homework 5

3. $a = 2mn$ $\quad b = m^2 - n^2$ $\quad c = m^2 + n^2$

Use $a - 120$ then

$2mn = 120$

$m \cdot n = 60$

possible (m, n) s.t. $b = m^2 - n^2 > 0$

$(60, 1) \rightarrow (120, 3599, 3601)$

$(30, 2) \rightarrow (120, 896, 904)$

$(20, 3) \rightarrow (120, 391, 409)$

$(15, 4) \rightarrow (120, 209, 241)$

$(12, 5) \rightarrow (120, 119, 169)$

$(10, 6) \rightarrow (120, 64, 136)$

Primitive triples (those that don't have a common divisor):

$(120, 209, 241)$ and $(120, 119, 169)$

and $(120, 3599, 3601)$

and $(120, 391, 409)$.

4. Prove that there are infinitely $a^2 + b^2 = c^2$ combinations.

Proof:

Take the simplest triples: $(3, 4, 5)$.

then any $n \cdot (3, 4, 5)$ will also be Pythagorean:

$(3n)^2 + (4n)^2 \stackrel{?}{=} (5n)^2$

$n^2 (3^2 + 4^2) \stackrel{?}{=} (5^2)\, n^2$

$3^2 + 4^2 \stackrel{\checkmark}{=} 5^2$. Therefore, it is necessary to have only one triple to see that there will be infinite combinations.

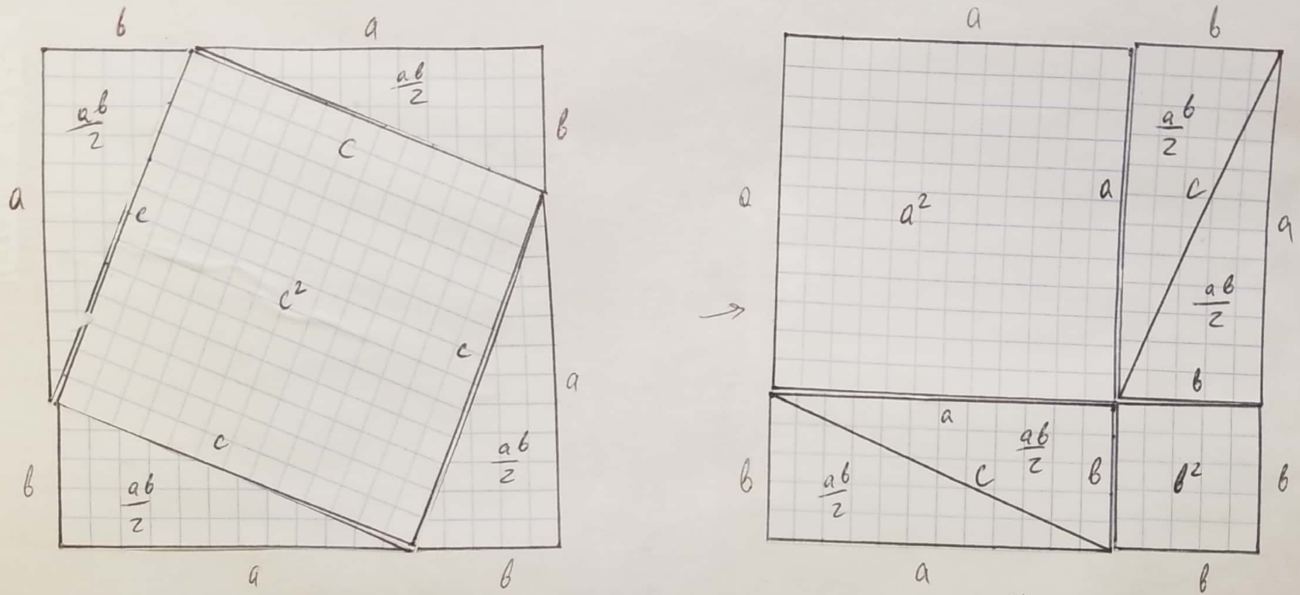
5. Fermat proved $a^4+b^4=c^4$ has no solutions ~~for~~ in positive integers.

6. Leonhard Euler proved that $a^3+b^3=c^3$ doesn't have positive solutions.

7. Fermat thought he had a proof for any integer $n \geq 3$, there are no solution in positive integers to $a^n+b^n=c^n$.

8. Andrew Wiles proved Fermat's last theorem in 1993.

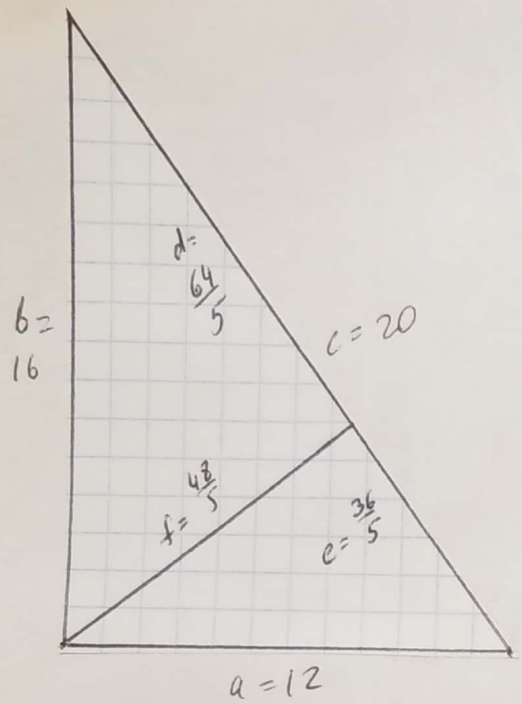
1.



$$c^2 + 4\left(\frac{ab}{2}\right) = a^2 + b^2 + 4\cdot\left(\frac{ab}{2}\right)$$

where $a = 12$, $b = 5$, $c = 13$

2.



1. $\frac{a}{c} = \frac{12}{20} = \frac{3}{5} = \frac{\frac{36}{5}}{12} = \frac{e}{a}$

$$(ac)\ \frac{a}{c} = \frac{e}{a}\ (ac)$$

$$a^2 = ec$$

2. $\frac{b}{c} = \frac{16}{20} = \frac{4}{5} = \frac{\frac{64}{5}}{16} = \frac{d}{b}$

$$(bc)\ \frac{b}{c} = \frac{d}{b}\ (bc)$$

$$b^2 = cd$$

3. Now add formulas from 1 and 2

$$a^2 + b^2 = ec + cd$$

$$a^2 + b^2 = c\,(e+d) = c \cdot c$$

$$a^2 + b^2 = c^2.$$

Chapter III

5. new period in Greek history

- upper class material existence based upon slavery
  - → leisure to cultivate arts and sciences
  - → relief from worry by studying philosophy
- Plato's Republic
  - → guards must study the quadrivium - arithmetic geometry astronomy and music
- Plato's academy - Archytas, Theaetetos and Eudoxos.
  - Theaetetos - theory of irrationals
  - Eudoxos - theory of proportions
    - ~exhaustion method - area + volume computations
    - ~solved the 'crisis' of Greek math
    - decide the course of Greek axiomatics
    - dismissed arithmetic theory of Pythagoreans
    - Dedekind + Weierstrass followed this mode of thought in developing irrational numbers.
    - Exhaustion method answered Zeno's paradoxes
      - use formal logic only
    - Axiom of Archimedes: magnitudes are said to have a ratio to one another which are capable, when multiplied, of exceeding one another.
    - quite rigorous proofs
- letter 'Method' from Archimedes to Eratosthenes
  - describes non-rigorous but fertile ways to discover results.
  - mathematical reasoning competing with Eudoxos

- Democritus's school of math reasoning
  - 'geometrical atom'
  - facilitated findings of new results.
  - not widely used
- Platonic idealism over Democritian materialism.

6. 334 B.C. Alexander the Great began conquering Persia
323 B.C - death
→ whole near East under Greece
→ emergance of 3 empires:
Egypt under Ptolemies
Mesopotamia + Syria under Seleucids
Macedonia under Antigonos
Spread of Hellenism - Greek culture over the world
→ monarch adopted Oriental manners
→ stimulated Greek arts, letters and sciences.
→ Egypt became central position of the Mediterranean world
→ greatest flowering of Hellenistic math
- Alexandria, Athens, Syracuse
educational center → Archimedes

7. Professional scientist - man who devoted his life to the pursuit of knowledge, receiving a salary
Alexandria, Egypt
→ learning center Museum with Library
→ Euclid - 13 books of 'Elements' ('stoicheia')
- 'Data' - applications of algebra to geometry

→ 'Elements' – heavily reproduced and studied book.
→ logical structure, school geometry
- logical deductions of theorems from definitions postulates and axioms

Books 1-4 – plane geometry
Book 5 – Eudoxos' theory of incommensurables
Book 6 – applied to similarity of triangles.
Book 10 – classification of quadratic irrationals and their quadratic roots: $\sqrt{a+\sqrt{b}}$
Books 11-13 – solid geometry
- discussion of five regular Platonic bodies
Book 7-9 – number theory
- Euclid's theorem: infinite numbers of primes
- Euclid's algorithm for greatest common divisor
- algebraic reasoning – geometric form
$\sqrt{A}$ – side of square with area A.

→ brought together Eudoxos' theory of proportions, Theaetetos' theory of irrationals and theory of five regular bodies.

8. Archimedes from Syracuse (287–212)
- adviser to King Hiero
- killed when Romans took over
- contributions to integral calculus
- 'Measurement of the Circle'

$3\frac{10}{71} < \pi < 3\,^1/_7$ — extending approximations to polygons of 96 sides

$\pi \approx 3\,^1/_7$

Archimedes (cont)

- 'On the Sphere and Cylinder'
  - expression for the area of the sphere is 4 times that of a great circle
  - expression for the volume of the sphere is 2/3 of circumscribed cylinder
- 'Quadrature of the Parabola'
- 'Spirals' - 'Spiral of Archimedes'
- 'On Floating Bodies' - a treatise on hydrostatics
- Axiom of Archimedes
- Cattle Problem leads to equation of 'Pell type':

$$t^2 - 4729494\,u^2 = 1$$

9. Apollonius of Perga (c.260–c.170)
   - treatise of 8 books on 'Conics'
     - ellipse, parabola and hyperbola
       $\to y^2 = px$ $\quad\to y^2 = px \pm \frac{p}{d}x^2$ plus - hyperbola, minus - ellipse
     - did not have a coordinate method
       - many results can be transcribed
   - tangency problem - construction of circles

10. Mathematics + astronomy
    - $\to$ agricultural $\Rightarrow$ importance of astronomy
    - structure of planetary system
      - easy, but stimulating math improvement
    - motion of the moon - most challenging problem
      - Greek + Babylonian math - advancement

- Planetary theory of Eudoxa
  - attempt to explain the motion of the planets by assuming super position of 4 rotating concentric spheres
    - (around the Earth
    - central ideas of all planetary theories until 17th Century
- Aristrachos of Samos (c. 250 BC)
  - the sun and not the earth is the center
  - wide acceptance that Earth rotates around its axis
  - small success of heliocentric hypothesis due to Hipparchos
- Hipparchos of Nicaea
  - observations 161-126 B.C. came to us from Ptolemy
  - 'Almagest' by Ptolemy
    - eccentric circles and epicycles to explain the motion of sun, moon and planets
  - lattitude and longitude by astronomical means

11. 212 . fall of Syracuse to Rome
146 fall of Carthage + Greece
64 fall of Mesopotamia
30 fall of Egypt
] Roman dominated Orient

- Western part - extensive agriculture fitted for slavery
  Eastern part - intensive agriculture, only domestic slaves
- spread of slave economy - ↓ interest in technical + scientific adjustments
  → some ruling class dabbled in science + math but promoted mediocrity
- Eastern sciences flourished (blend of Hellenistic and Oriental elements)

- pax Romano - lasting for many centuries
  - pax Sinensis - Eurasian continent
  - diffusion of knowledge over the continent
  - spread of sexagesimal division of angle and hour
- decline of Greek mathematics
  - clumsy geometrical mode of expression
  - consistent rejection of algebraic notation
- Egyptian-Babylonian math
  - abstract geometrical demonstrations

12. Nichomacos of Gerasa (A.D 100) - Alexandrian mathematician
- 'Arithmetic Introduction'
  - ~ similar to Euclid's elements
  - use of arithmetical notation

Ptolemy's 'Great Collection'
- astronomical opus
- trigonometry - table of chords (sine table)
  - → for 1° → $(1, 2.50) = \frac{1}{60} + \frac{2}{60^2} + \frac{50}{60^3} = 0.0172$.
  - → for π → $(3, 8, 30) = 3.14166$

Ptolemy's theorem for quadrilateral inscribed in a circle
Ptolemy's Planisphaerium - stereographic projection and of latitude and longitude
Menelaos 'Sphaerica'
- purely Euclidean geometry.

Heron - lunar eclipse of 62 A.D.
- geometrical computational and mechanical
- 'Metrica' → 'Heronic' formula $A = \sqrt{s(s-a)(s-b)(s-c)}$

Heron's 'Metrica'
- Egyptian unit fractions
  $$\sqrt{63} = 7 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$$
- formula for pyramid reduced to Moscow Papyrus

13. Diophantus' 'Arithmetica' (c. 250 AD)
- only 6 books survive
- indeterminate equations
- 'Diophantine analysis'
  $$Ax^2 + Bx + C = y^2$$
  $$Ax^3 + Bx^2 + Cx + D = y^2$$
  - only interested in positive rational solutions
  Pell equations:
  $$x^2 - 26y^2 = 1 \quad \text{and} \quad x^2 - 30y^2 = 1$$
- first systematic use of algebraic symbols
  - special sign for unknown, minus, reciprocals

14. Pappos' 'Collection' ('Synagoge')
- handbook to study of Greek geometry
- results of ancient authors
  - problems inspired much later research

Alexandrian school died with decline of antique society
Proclus (410-485), 'Commentary on the First Book of Euclid'
- Neoplatonist school in Athens

Academy in Athens discontinued as pagan by the Emperor Justinian (529)
630 - Alexandria conquered by the Arabs.

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15. Greek arithmetic and logistics
- 'arithmetica', science of numbers ('arithmoi')
- 'logistics' - practical computation
- method of numeration - additive decimal principle
  - successive symbols of the Greek alphabet to express 1-9, then 10-90 tens and hundreds
  - decimal non-position system
    - both ιδ and δι could mean 14
  - use no less than 27 symbols but
    - many mathematicians used it with ease
    - accepted by Greek merchants
    - long persistence