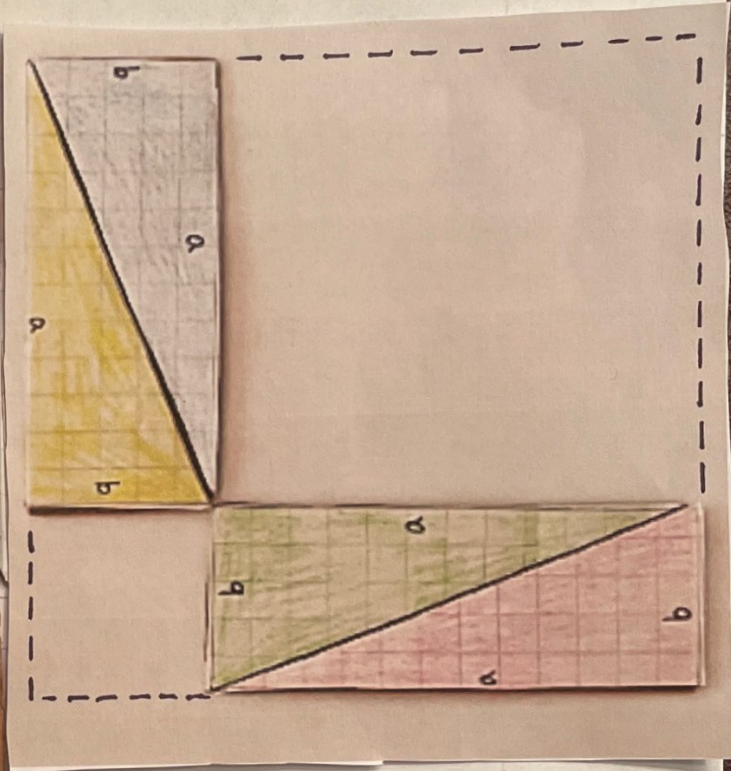
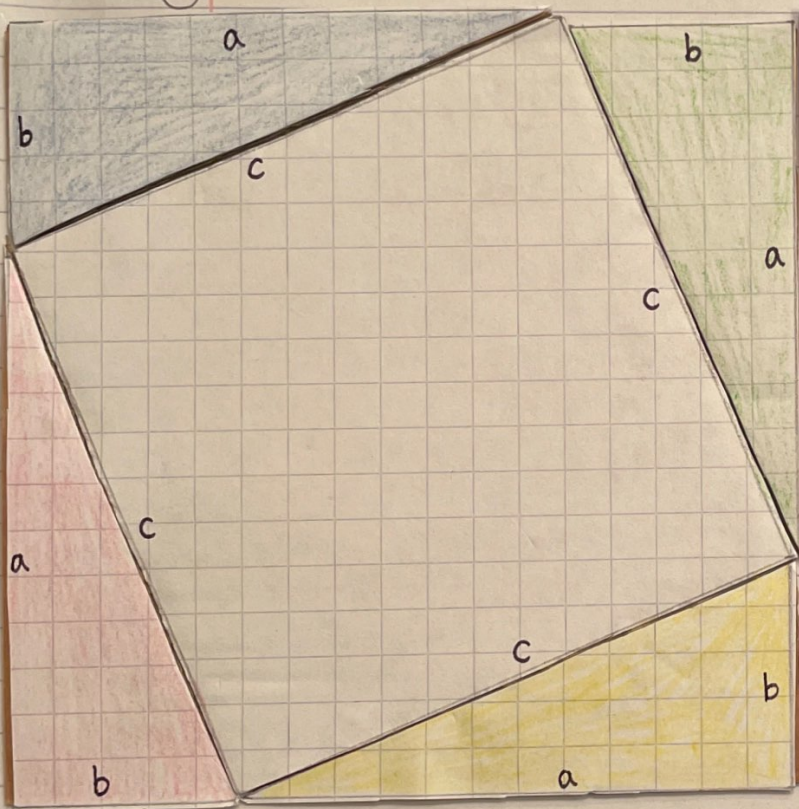


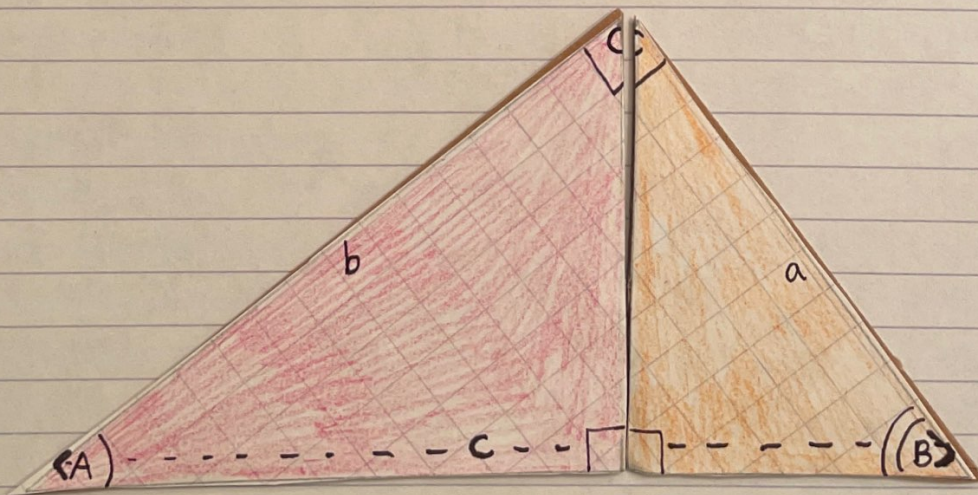
Sarah Magno
Dr. Z, History of Math
9/26/21

Homework for Lecture 5 - OK to post

①



②



$$(3) \quad a = 120 = 2mn \quad b = m^2 - n^2 \quad c = m^2 + n^2$$

Since $120 = 2mn$, then $60 = mn$, so m and n are factors of 60. So the possible combinations of m and n are

m	n	$a = 120$	$b = m^2 - n^2$	$c = m^2 + n^2$
60	1	$a = 120$	$b = 60^2 - 1^2 = 3599$	$c = 60^2 + 1^2 = 3601$
30	2	$a = 120$	$b = 30^2 - 2^2 = 896$	$c = 30^2 + 2^2 = 904$
20	3	$a = 120$	$b = 20^2 - 3^2 = 391$	$c = 20^2 + 3^2 = 409$
15	4	$a = 120$	$b = 15^2 - 4^2 = 209$	$c = 15^2 + 4^2 = 241$
12	5	$a = 120$	$b = 12^2 - 5^2 = 119$	$c = 12^2 + 5^2 = 169$
10	6	$a = 120$	$b = 10^2 - 6^2 = 64$	$c = 10^2 + 6^2 = 136$

So the Pythagorean triples are

(120, 3599, 3601)	primitive because $\gcd(120, 3599, 3601) = 1$
(120, 896, 904)	
(120, 391, 409)	primitive because $\gcd(120, 391, 409) = 1$
(120, 209, 241)	primitive because $\gcd(120, 209, 241) = 1$
(120, 119, 169)	primitive because $\gcd(120, 119, 169) = 1$
(120, 64, 136)	

④ Let m and n be positive integers such that $m > n > 0$. Then the integers a, b , and c can be written as

$$a = m^2 - n^2 \quad b = 2mn \quad c = m^2 + n^2$$

We will show that $c^2 - a^2 - b^2 = 0$, so that $a^2 + b^2 = c^2$. We rewrite $c^2 - a^2 - b^2$ as

$$\begin{aligned} c^2 - a^2 - b^2 &= (m^2 + n^2)^2 - (m^2 - n^2)^2 - (2mn)^2 \\ &= m^4 + 2m^2n^2 + n^4 - m^4 + 2m^2n^2 - n^4 - 4m^2n^2 \\ &= 0 \end{aligned}$$

Since $c^2 - a^2 - b^2 = 0$, then $a^2 + b^2 = c^2$.

⑤ Fermat

⑥ Euler

⑦ Fermat

⑧ Andrew Wiles