

Sarah (Ok to post)

Homework 5

9/26/2021

3.  $a = 2mn, b = m^2 - n^2, c = m^2 + n^2, a = 120$

$$a^2 + b^2 = c^2$$

$$120^2 + (m^2 - n^2)^2 = (m^2 + n^2)^2$$

$$14400 + m^4 + 2m^2n^2 + n^4 = m^4 + 2m^2n^2 + n^4$$

$$14400 = 4m^2n^2 \quad m^2n^2 = 3600$$

$$b = mn = 60$$

Let  $m = 20, n = 3 : a = 120, b = 39, c = 409$

$$m = 10, n = 6 : a = 120, b = 64, c = 136$$

$$m = 60, n = 1 : a = 120, b = 3599, 3601 = c$$

$$m = 15, n = 4 : a = 120, b = 209, 241 = c$$

$$m = 30, n = 2 : a = 120, b = 896, 904 = c$$

$$m = 12, n = 5 : a = 120, b = 119, c = 169$$

The primitive triples are:  $(120, 39, 409), (120, 64, 136), (120, 3599, 3601), (120, 209, 241)$

4.  $a^2 + b^2 = c^2$  Let  $a, b, c$  be integers.

Since  $a, b, c$  are all integers,  $a^2$  and  $b^2$  would be integers.

Also, since  $a^2$  and  $b^2$  are integers, their sum  $a^2 + b^2$  is an integer.

Similarly, since  $c$  is an integer,  $c^2$  would be an integer. So, there must be infinitely many integers such that  $c^2 = a^2 + b^2$ .

5. Pierre de Fermat

6. Leonhard Euler

7. Pierre de Fermat

8. Andrew Wiles