History of Math | Homework 5 | Karan Amin

It's okay to post this on website

3

$a = 2mn$

$b = m^2 - n^2$

$c = m^2 + n^2$

$a = 120$

$a = 120 = 2mn$

$60 = mn$ Factors of 60 [1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60]

$m = 60$, $n = 1$, $a = 120$, $b = 3599$, $c = 3601$

$m = 30$, $n = 2$, $a = 120$, $b = 896$, $c = 904$

$m = 20$, $n = 3$, $a = 120$, $b = 391$, $c = 409$

$m = 15$, $n = 4$, $a = 120$, $b = 209$, $c = 241$

$m = 12$, $n = 5$, $a = 120$, $b = 119$, $c = 169$

$m = 10$, $n = 6$, $a = 120$, $b = 64$, $c = 136$

All of them are primitive except the last one. a, b, c are multiples of 8. The last triples are multiples of (15, 8, 17)
a, b, c

Scratch Work

$m = 60$, $n = 1$

$b = 60^2 - 1 = 3599$

$c = 60^2 + 1^2 = 3601$

$m = 30$, $n = 2$

$b = 30^2 - 2^2 = 896$

$c = 30^2 + 2^2 = 904$

$m = 20$, $n = 3$

$b = 20^2 - 3^2 = 391$

$c = 20^2 + 3^2 = 409$

$m = 15$, $n = 4$

$b = 15^2 - 4^2 = 209$

$c = 15^2 + 4^2 = 241$

$m = 12$, $n = 5$

$b = 12^2 - 5^2 = 119$

$c = 12^2 + 5^2 = 169$

$m = 10$, $n = 6$

$b = 10^2 - 6^2 = 64$

$c = 10^2 + 6^2 = 136$

4 We know there exists at least one pythagorean triple. lets call it (a_1, b_1, c_1). We know that $a_1, b_1, c_1 \in \mathbb{Z}$ and $a_1^2 + b_1^2 = c_1^2$. Let k be any natural number, $k \in \mathbb{N}$

Then $(ka_1)^2 + (kb_1)^2 = k^2 a_1^2 + k^2 b_1^2 = k^2(a_1^2 + b_1^2) = k^2 \cdot c_1^2 = (kc_1)^2$

Because $ka_1, kb_1, kc_1 \in \mathbb{Z}$, we have that (ka_1, kb_1, kc_1) is another triplet. Because k can be any natural number, and the set of natural numbers is infinite, the set of triples is infinite too. ■

5 Pierre de Fermat

6 Leonhard Euler

7 Pierre de Fermat

8 Andrew Wiles