Hw5

Wednesday, September 29, 2021 1:41 AM

3. Using $a = 2mn, b = m^2 - n^2, c = m^2 + n^2$, find as many Pathagorean triples as you can with $a = 120$. Which ones are **primitive**?

$a = 120$

$120 = 2mn$

$60 = mn$

$1, 2, 3, 4, 5, 6$

$60, 30, 20, 15, 12, 10$

$b = m^2 - n^2$

$b = 60^2 - 1^2 = 3599$

$= 30^2 - 2^2 = 896$

$= 20^2 - 3^2 = 391$

$= 15^2 - 4^2 = 209$

$= 12^2 - 5^2 = 119$

$= 10^2 - 6^2 = 64$

$c = m^2 + n^2$

$c = 60^2 + 1^2 = 3601$

$= 30^2 + 2^2 = 904$

$= 20^2 + 3^2 = 409$

$= 15^2 + 4^2 = 241$

$= 12^2 + 5^2 = 169$

$= 10^2 + 6^2 = 136$

a	b	c
$120^2 +$	$3599^2 =$	3601^2
$120^2 +$	$896^2 =$	904^2
$120^2 +$	$391^2 =$	409^2
$\vdots +$	$209^2 =$	241^2
$\vdots +$	$119^2 =$	169^2
$\vdots +$	$64^2 =$	136^2

m	n
✓	[60, 1]
✓	[30, 2]
✓	[20, 3]
✓	[15, 4]
✓	[12, 5]
✓	[10, 6]

→ Pythagorean Triples

Primitive

a	b	c
120	3599	3601
120	391	409
120	209	241
120	119	169

4. Prove that there are infinitely triples of **positive** integers a, b, c such that

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$$a^2 + b^2 = c^2 \quad .$$

Consider $n^2 + 2n + 1 = (n+1)^2$

- when $2n+1$ is a square, a pythagorean triple is formed.
- $2n+1$ is every odd number, every other square number is odd, and there are infinite odd numbers. Thus, there are an infinite number of pythagorean triples

5. Who first proved that there are no solutions, in **positive** integers a, b, c, of the equation

$$a^4 + b^4 = c^4 \quad .$$

Pierre de Fermat

6. Who first proved that there are no solutions, in **positive** integers a, b, c, of the equation

$$a^3 + b^3 = c^3 \quad .$$

Pierre de Fermat

7. Who first *thought* that he has a proof that, for any integer $n \geq 3$, there are no solutions, in **positive** integers a, b, c, of the equation

$$a^n + b^n = c^n \quad ?$$

Pierre de Fermat

8. Who first actually had a proof that, for any integer $n \geq 3$, there are no solutions, in **positive** integers a, b, c, of the equation

$$a^n + b^n = c^n \quad ?$$

Andrew Wiles

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