Nabhanya Neb Homework 4

1. A travels 200 mph
T travels 50 mph, gets 50 miles headstart

$D(A) = 200t$
$D(T) = 50 + 50t$

(a) at $t=0$: $D(A) = 0$ $\quad D(T) = 50$
$t = 1/4$: $D(A) = 50$ $\quad D(T) = 50 + 50/4 = 250/4$
$t = 5/16$: $D(A) = 250/4$ $\quad D(T) = 250/4 + 50/16 = 1050/16$

$\vdots$

According to Zeno's proof, we can keep incrementing t, but by the time A catches up to T's old distance, T will be a bit ahead.

(b) $200t = 50 + 50t$
$150t = 50$
$t = 1/3$

(c) $\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$ (or $\frac{1}{4} + \frac{1}{16} + \cdots$)

since $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, we have our sum to be $\frac{1}{4}\left(\frac{1}{3/4}\right) = \left(\frac{1}{4}\right)\left(\frac{4}{3}\right) = \boxed{1/3}$

(d)

| $t$ | $D(A)$ | $D(T)$ |
| --- | --- | --- |
| 0 | 0 | 50 |
| $1/4$ | 50 | $50 + 50/4$ |
| $5/16$ | $50 + 50/4$ | $50 + 50/4 + 50/16$ |
| $21/64$ | $50 + 50/4 + 50/16$ | $50 + 50/4 + 50/16 + 50/64$ |

No longer possible to sub-divide time
T is ahead

(2) prove that $1 + x + x^2 + \dots + x^n = \dfrac{1 - x^{n+1}}{1-x}$

$$(1-x)\left[1 + x + x^2 + \dots + x^n\right] = 1 - x^{n+1}$$

$$\Rightarrow (1-x) + x(1-x) + x^2(1-x) + \dots + x^n(1-x) = 1 - x^{n+1}$$

$$1 \underbrace{- x + x}_{0} \underbrace{- x^2 + x^2}_{0} - x^3 + \dots \underbrace{- x^n + x^n}_{0} - x^{n+1} = 1 - x^{n+1}$$

$$1 - x^{n+1} = 1 - x^{n+1} \checkmark$$

(3) $\displaystyle\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ if $0 \leq x < 1$

(a) $\displaystyle\sum_{i=0}^{i=k} x^i = \frac{1 - x^{k+1}}{1-x}$

If $k = \infty$, $x^k$ will be an infinitely small value. The numerator will be 1 so $\displaystyle\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

(b) $D(T) = 1 + Xt$

$D(A) = t$

$t = 0$ $\quad D(A) = 0$ $\quad D(T) = 1$

$t = 0 + X$ $\quad D(A) = \frac{1}{X}$ $\quad D(T) = 1 + \frac{1}{X}$

$t = 0 + X^2$ $\quad D(A) = \frac{1}{X} + \frac{1}{X^2}$ $\quad D(T) = 1 + \frac{1}{X} + \frac{1}{X^2}$

$\vdots$

I $\quad t = 0 + X^2 + \dots + X^n$ $\quad D(A) = \frac{1}{X} + \frac{1}{X^2} + \dots \frac{1}{X^n}$ $\quad D(T) = 1 + \frac{1}{X} + \frac{1}{X^2} + \dots + \frac{1}{X^2}$

II

$$1 + Xt = t$$
$$1 = t - Xt$$
$$1 = t(1 - X)$$
$$t = \frac{1}{1 - X}$$

(4)

$$\frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^n} = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1$$

If we keep splitting in half, we will never reach an end.

Thus, $\frac{1}{2} = \frac{1}{4} + \dots + \frac{1}{2^n}$

$$\frac{1}{2} + \frac{1}{2} = 1$$

5 $\begin{bmatrix} n & a & b & h & a \\ a & b & l & e & s \\ b & l & i & n & k \\ h & e & n & c & e \\ a & s & k & e & d \end{bmatrix}$ is my favorite :)