History of Math | Homework 4 | Karan Amin | It's okay to post this on website

1 Car A rides at 200 mph | T gets 50 mile head start
Car T rides at 50 mph

a) $t=0$

* $dist(A)=0$, $dist(T)=50$

It takes A $\frac{50}{200}$ = 1/4 hours to get to T's start position, by then T made it to $50+\frac{1}{4}(50) = 50+12.5 = 62.5$

At $t = 1/4$ hours

* $dist(A) = 50$, $dist(T) = 62.5$

Then $\frac{62.5-50}{200} = 0.0625$ hours for A to get to 62.5 miles. In the same time T traveled $0.0625 \cdot 50 = 3.125$. Total $62.5+3.125 = 65.625$

$t = 0.25+0.0625 = 0.3125$ hours

* $dist(A) = 62.5$ miles, $dist(T) = 65.625$ miles

Then $\frac{65.625-62.5}{200} = 0.015625$ $\quad$ $0.015625 \cdot 50 = 0.78125$

$65.625+0.78125 = 66.40625$

$t = 0.3125+0.015625 = 0.328125$

$dist(A) = 65.625$ $\quad$ $dist(T) = 66.40625$

⋮

We continue on forever, as soon as Car A gets to where Car T was in the last time step, Car T has moved on a little bit further in the same period of time.

b) $200t = 50+50t$

$\downarrow$

$150t = 50 \rightarrow t = \frac{50}{150} = \frac{1}{3}$ hours

Car A will catch up to Car T in 1/3 hours

c) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots = \frac{1}{4}\left(1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}\right) = \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$

$a = 1/4$

$r = 1/4 < 1$

$= \frac{1/4}{1 - 1/4} = \frac{1/4}{3/4} = \boxed{\frac{1}{3}}$ hours

$= \frac{a}{1-r}$

d) $t = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} = \frac{16 + 4 + 1}{64} = \frac{21}{64}$

$\text{dist}(A) = \frac{21}{64} \cdot 200 = \frac{525}{8}$

$= 65.625$

$\text{dist}(T) = 50 + \frac{21}{64} \cdot 50 = \frac{2125}{32}$

$= 66.40625$

$t = \frac{21}{64} + \frac{1}{64} = \frac{22}{64}$

$\text{dist}(A) = \frac{22}{64} \cdot 200 = 68.75$, $\text{dist}(T) = 50 + \frac{22}{64} \cdot 50 = 67.1875$

A passed T!

$\boxed{2}$ $1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}$

$\underline{\underline{x \neq 1}}$ Important

$(1-x)(1 + x + x^2 + \cdots + x^n) = 1 - x^{n+1}$

$(1-x) + x(1-x) + x^2(1-x) + \cdots + x^n(1-x) = 1 - x^{n+1}$

$(1-\cancel{x}) + (\cancel{x} - \cancel{x^2}) + (\cancel{x^2} - \cancel{x^3}) + (\cancel{x^3} - x^4) + \cdots + (\cancel{x^n} - x^{n+1}) = 1 - x^{n+1}$

$1 - x^{n+1} = 1 - x^{n+1}$ ✓

True

3 If $0 < x < 1$ then

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

a) Using ~~two~~ 2

Since $0 < x < 1$, then for $n > 1$ $\lim_{n\to\infty} x^n = 0$

$$\sum_{n=0}^{\infty} x^n = \lim_{n\to\infty} \frac{1-x^{n+1}}{1-x} = \frac{1-0}{1-x} = \frac{1}{1-x}$$

b) $A \to 1$ mps

$T \to x$ mps $\quad x < 1$ | T head start 1 mile

$t = 0$

* $dist(A) = 0$, $dist(T) = 1$

$\frac{1-0}{1} = 1 \qquad 1 \cdot x = x$

$t = 1$

* $dist(A) = 1$, $dist(T) = 1 \cdot x = x$

$\frac{x-1}{1} = x-1 \qquad (x-1)\cdot x = x^2 - x \qquad x^2 + x - x = x^2$

$t = 1 + (x-1) = x$

* $dist(A) = x$, $dist(T) = x^2$

$x^2 - x \qquad (x^2-x)\cdot x = x^3 - x^2 \qquad x^3 - x^2 + x^2 = x^3$

$t = x + x^2 - x = x^2$

* $dist(A) = x^2$, $dist(T) = x^3$

Zeno $\hookrightarrow$ meeting time

$1 + x + x^2 + \dots = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad a = 1,\ r = x$

Algebra Way

$$\sum_{n=0}^{\infty} x^n = S$$

$1 + x + x^2 + \dots = S$

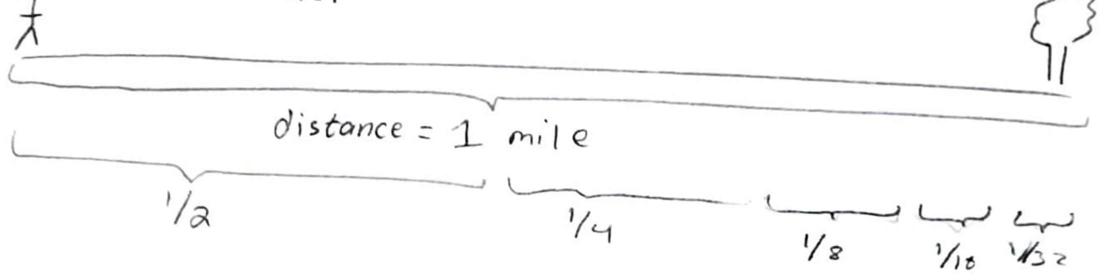
$x(x^{-1} + 1 + x^1 + \dots) = x(x^{-1} + S) = S$

$1 + xS = S \Rightarrow 1 = S - xS \Rightarrow 1 = S(1-x)$

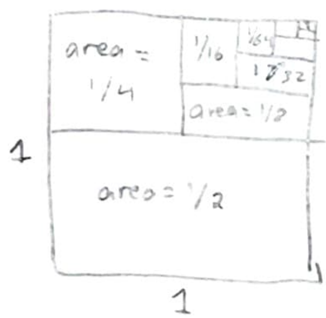
$$\boxed{S = \frac{1}{1-x}}$$

4 Dichotomy paradox

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$




Before zeno can travel the whole 1 mile, he must travel half way. Then he again travel half that distance again 1/4 of the way. He keeps doing this forever




Area of square = $1 \times 1 = 1$

Area of sub sections

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

They must be equal hence

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$