HW 4

1. a) $50 \div 200 = \frac{1}{4}h$

$(50 \times \frac{1}{4}) \div 200 = 0.0625h = \frac{1}{16}h$

$(50 \times 0.0625) \div 200 = 0.015625h = \frac{1}{64}h$

so A will never catch up T

b) $50 \div (200-50) = \frac{1}{3}h$

c). Total time is $f(n) = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots + \frac{1}{4^n} = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{3}$

2. when $n=0$ $1 = \frac{1-x^1}{1-x} = 1$ correct

Assume $n=k$ is correct

so $1 + x + \cdots + x^k = \frac{1-x^{k+1}}{1-x}$

when $n = k+1$

$$1 + x + \cdots + x^k + x^{k+1} = \frac{1-x^{k+1}}{1-x} + x^{k+1} = \frac{(1-x^{k+1}) + (1-x)x^{k+1}}{1-x}$$

$$= \frac{1-x^{k+1}+x^{k+1}-x^{k+2}}{1-x} = \frac{1-x^{k+2}}{1-x}$$

so the statement is correct

3. a) $\sum_{n=0}^{\infty} x^n = 1 + \cdots + x^n = \frac{1-x^{n+1}}{1-x}$

Ass $n \to \infty$ $x^{n+1} \to 0$ because $0 < x < 1$

so $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ as $n \to \infty$

b) first: $\frac{1}{1} = 1s$ $\frac{x}{1} = xs$

$\frac{x^2}{1} = x^2 s$ $\cdots$ $\sum_{n=0}^{x} x^n s$

second: $\frac{1}{1-x}$

4. $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$