Sarah Cutbric (ok to post) Homework 4 9/26/2021

1) i) A's speed is 200 miles per hour; T's speed is 50 miles per hour
T gets a head start of 50 miles

a. At time 0, T is at 50 and A is at 0. At $t = \frac{1}{4}$ of an hour, A has moved up to 50, but T has also moved up to $50 + \frac{1}{4}(50)$. At $t = \frac{1}{4} + \frac{1}{16}$, A has moved up to 62.5, but T has also moved up to $50 + \frac{1}{4}(50) + \frac{1}{16}(50)$. At $t = \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$, A has moved up to $50 + \frac{1}{4}(50) + \frac{1}{16}(50)$, but T has also moved to $50 + \frac{1}{4}(50) + \frac{1}{16}(50) + \frac{1}{64}(50)$. Wherever A goes to where T was, T moves up aswell, so A will never be able to reach T.

b) After t hours: A's distance $= 200t$

T's distance $= 50t + 50$

A's distance = T's distance when $200t = 50t + 50$

$150t = 50 \Rightarrow t = \frac{1}{3}$ of an hour

so A will catch up to T after 20 minutes

c) $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^{n+1}$

so the sum would be $\frac{1/4}{1 - 1/4} = \frac{1/4}{3/4} = \frac{1}{3}$ so A would catch up to T after 20 minutes

d) At time $t = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} = \frac{21}{64}$, A is at $\frac{525}{8}$ and T is at $\frac{2125}{32}$.
The next step would require $\frac{1}{256}$ of an hour, but since time advances by $\frac{1}{64}$, the next step would be

2) $1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}$

$(1-x)(1 + x + x^2 + \cdots + x^n) = 1 - x^{n+1}$

$1 + \cancel{x} + \cancel{x^2} + \cdots + \cancel{x^n} - \cancel{x} - \cancel{x^2} \cdots - x^{n+1} = 1 - x^{n+1} = 1 - x^{n+1}$

The middle terms ($x, x^2, x^3, \ldots, x^n$ are all subtracted) so you're only left with 1 and $-x^{n+1}$ on the left side, which is equal to what's on the right side.

3) a. $1+x+x^2+\cdots+x^n=\frac{1}{1-x}$

$(1-x)(1+x+x^2+\cdots+x^n)=1$

$1+\cancel{x}+\cancel{x^2}+\cdots+\cancel{x^n}-\cancel{x}-\cancel{x^2}-\cdots-x^n=1$

On the left side, the only terms you are left with is 1 because through multiplication the other terms cancel out so, this proves that the sum works.

b. Zeno's way: At $t=0$, T is at 1 and A is at 0. At $t=1$, A is at 1, but T has gone $1+x$. At $t=1+x$, A has reached where T was, but T has moved up to $1+x+$

Algebraic way: A's distance $=1t$ T's distance $=1+xt$

$1+xt=t$

$1=(1-x)t$ $t=\frac{1}{1-x}$

4) $\sum_{n=1}^{\infty}\frac{1}{2^n}=1$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}=1$

I'm not sure how to prove this using Zeno's paradox because Zeno supposed that you'd never reach the destination.